Informative fundraising: The signaling value of seed money and matching gifts *

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Abstract

While existing theory predicts that matching leadership gifts raise more voluntary contributions for public goods than seed money, recent experiments find otherwise. We reconcile the two by studying a model of leadership giving with incomplete information about the quality of the public good provided by a charity. Both the fundraising scheme employed by the charity and the contribution decision by the lead donor may signal the charity's quality to subsequent donors. The charity solicits optimally for a matching gift if the lead donor is informed about the quality of the public good. Intuitively, an informed lead donor conveys quality information to downstream donors through the size of her contribution. As a result, the charity has no signaling concerns and opts for matching because it mitigates the free-riding incentives among donors and leads to higher contributions. The preference for matching, however, reduces when the lead donor's information is limited. Then, the lead donors contribution is less informative and the high quality charity utilizes the fundraising scheme to convey information. In particular, the charity uses seed money as a costly signaling device to convince donors of its high quality. As a result, seed money becomes a strong signal of quality and induces higher expected contributions by donors.

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1 Introduction

Leadership giving has garnered significant attention in the charitable giving literature (e.g., Vesterlund, 2003; Andreoni, 2006; List and Lucking-Reiley, 2002; Rondeau and List, 2008; Huck et al., 2015).[1] It refers to a fundraising strategy by charities which entails soliciting first a wealthy donor and announcing his or her donation to encourage others' giving. Most commonly, leadership gifts are in the form of an unconditional lump sum donation called "seed money" or a promise of matching small donations by a fixed ratio called "matching gift".

Theoretical studies suggest that a matching gift should encourage more donations than seed money since the former strategy reduces free-riding (e.g., Varian, 1994; Guttman, 1978; Danziger and Schnytzer, 1991). Recent experimental studies, however, find otherwise. In particular, Rondeau and List (2008), Karlan et al. (2011), Huck and Rasul (2011), and Huck et al. (2015) find evidence that a seed money announcement attracts more donors and increases total contributions, while a matching gift announcement has a weak or even adverse effect on contributions.

In this paper, we reconcile this ostensible discrepancy between the theory and the experimental evidence. Our theoretical argument relies on the observation that the choice of the fundraising scheme–seed or matching–conveys information to donors about the charity's quality. We find that seed money, which is seemingly the less effective strategy, can be used by the charity to credibly signal higher quality. This explains the significant increase in giving upon an announcement of a seed money gift and the weak response to a matching gift.

Our model features a large economy with altruistic donors, in which the charity is privately informed about the quality of the public good that it provides.[2] Donors possess limited information about the quality of the public good. This setting is consistent with the state of the non-profit sector in the USA that features significant quality heterogeneity among existing non-profits and limited information among donors about how non-profits use their donations.[3] In our base model, this quality is binary, but we extend our analysis in Section

[1]On the theory front, other related studies include Varian (1994), Guttman (1978), Danziger and Schnytzer (1991), Admati and Perry (1991), Andreoni (1998), Romano and Yildirim (2001), Bag and Roy (2011), Krasteva and Yildirim (2013), Gong and Grundy (2014). There is a large experimental literature that includes both lab experiments (e.g., Eckel and Grossman, 2003; Potters et al., 2005; Güth et al., 2007; Potters et al., 2007; Eckel and Grossman, 2006a; Eckel and Grossman, 2006b; Eckel et al., 2007) and field experiments in a variety of settings (e.g., Silverman et al., 1984; Frey and Meier, 2004; Meier and Frey, 2004; Soetevent, 2005; Meier, 2007; Falk, 2007; Alpizar et al., 2008; Croson and Shang, 2008; Shang and Croson, 2009; Karlan and List, 2007; Eckel and Grossman, 2008; Karlan et al., 2011; Huck and Rasul, 2011; Adena and Huck, 2017).

[2]While our main analysis focuses on purely altruistic donors, Section 5.3 extends the analysis to impure altruism in line with Andreoni (1988, 1990).

[3]For instance, Charity Navigator, the largest charity rating agency in the USA, has classified close to one-third of rated charities in years 2007-2010 as having exceptionally poor or poor performance (Yörük, 2016). At the same time, the Money for Good 2015 survey finds that "49% [of donors] don't know how nonprofits use their money."

5.1. The charity chooses its fundraising mechanism to maximize donations. In particular, the charity chooses whether to solicit the lead donor for seed money or a matching gift. Subsequently, given this fundraising strategy, the lead donor decides whether to acquire costly information about the public good's quality before making a donation decision. Under leadership giving, the information acquired not only benefits the lead donor directly, as it results in more informed giving, but it enables the lead donor to signal the quality of the public good to downstream donors through the size of her contribution.

In general, both the fundraising strategy and the size of the lead donor's gift may convey information about the charity's quality. These two signaling channels interact with each other to form the charity's and the lead donor's equilibrium strategies. In particular, the lead donor acquires information only if the value of information exceeds the cost. The value of information, however, varies not only with the prior quality distribution, but also with the equilibrium fundraising strategy. Thus, multiplicity of equilibria may arise with varying degree of information acquisition.

Considering first the two extremes of fully informed and fully uninformed equilibria, we establish that these types of equilibria cannot explain the use of seed money. The first extreme of fully informed lead donor causes the charity to rely on the lead donor to reveal the charity's quality to subsequent donors through the size of her donation. This eliminates the signaling concern of the high quality charity when choosing her optimal fundraising scheme. Consequently, consistent with the existing theoretical literature, we find that in the absence of signaling considerations, the charity optimally chooses the matching scheme, as it alleviates the free-riding problem present in public good provision. The other extreme of fully uninformed lead donor also fails to explain the success of seed money over matching. This is because in the absence of quality verification by the lead donor, the low quality charity can costlessly imitate the high quality charity's fundraising strategy. This makes it impossible for the high quality charity to separate from its low quality counterpart. Thus, donors fail to learn any useful information from the fundraising scheme or the lead donor's contribution amount and as a result all charity types and all schemes on the equilibrium path raise the same amount of money.

In order to explain the successful use of seed money, we turn to partially informed equilibria with seed money on the equilibrium path. We refer to such equilibria as *SPI* (seed-partial info) equilibria. Note that under partial information acquisition by the lead donor, imitation by the low quality charity is not costless any longer since there is some possibility of

At first glance, this lack of information might be attributed to the donors' lack of interest. However, the survey finding suggests otherwise. It reveals that donors "want clearer communication with nonprofits" regarding the charitable services that their money provides. The lack of information is attributed to the fact that "[donors] are often uncertain where to start, don't have the information they want, feel pressed for time, ...". For the full survey conducted by Camber Collective, visit `http://www.cambercollective.com/moneyforgood/`.

verification. Moreover, seed money is the less efficient scheme as it results in lower overall contributions compared to matching for any fixed quality level. Consequently, seed money is used as a costly quality signal. In particular, we show that in *every SPI* equilibrium, the high quality charity chooses seed money fundraising more frequently compared to the low quality charity, causing seed money to emerge as a signal of higher quality. Intuitively, as the lead donor becomes less reliable at signaling quality, the high quality charity engages in costly signaling through the fundraising scheme by choosing to solicit for seed money.

Our theoretical analysis establishes a plausible mechanism by which leadership giving may convey information to donors. Section 5 demonstrates the robustness of seed money as a signal of higher quality in richer economic environments that include large number of quality types (Section 5.1), the possibility of opting out of leadership giving (Section 5.2), the presence of warm-glow motivations for giving (Section 5.3), and the availability of alternative information channels (Section 5.4). Section 5.1 reveals that while the likelihood of seed money fundraising is not necessarily monotonically increasing in the charity's quality, seed money is associated with higher expected quality in *every SPI* equilibrium. Section 5.3 establishes that altruistic motives for giving are important for incentivizing quality signaling via the fundraising scheme since strong warm-glow motivations for giving cause donors to become less sensitive to the scheme choice, thus making both schemes equally attractive for the charity. In general, our analysis suggests that the use of seed money as a costly signaling device persists in the presence of sufficiently strong altruistic motives for giving and significant quality uncertainty that is not fully resolved by the lead donor's information acquisition strategy or alternative information channels. This suggests that seed money fundraising is likely a more attractive fundraising strategy for newer charities with significant public good component, who are striving to establish quality reputation among donors.

Related Literature Our theoretical model builds upon a large theoretical literature. Early theoretical work on private provision of public goods, such as Warr (1983) and Bergstrom et al. (1986), have focused on simultaneous contributions. They show the equivalence of the non-cooperative equilibrium from the simultaneous contributions game to the Lindahl equilibrium. Admati and Perry (1991) expand the analysis to a mechanism of alternating sequential contributions towards a threshold public good. They find that this can lead to an inefficient outcome. Similarly, Varian (1994) considers sequential fundraising and finds that it results in lower public good provision compared to simultaneous contributions due to donors' incentives to free-ride on earlier contributions. However, the possibility of donors subsidizing each others' contributions can alleviate this problem (e.g. Guttman, 1978; Danziger and Schnytzer, 1991). The implication of these findings is that a matching gift is more effective at encouraging contributions by downstream donors compared to a seed money gift.

In the context of complete and symmetric information, the use of seed money can be rationalized by the presence of a threshold public good or other-regarding preferences. In particular, Andreoni (1998) shows that charities can use seed money to avoid zero-contributions equilibrium, in which no donor contributes due to an expectation that the threshold will not reached. Romano and Yildirim (2001) show that other-regarding preferences can give rise to upward-sloping best response functions, making sequential fundraising more effective than simultaneous fundraising. In the context of standard altruistic preferences, Gong and Grundy (2014) illustrate the possibility of matching raising less donations than seed money due to the lead donors' reluctance to offer high match ratios. They show that a necessary (but not sufficient) condition for this is that donors' marginal utility of the public good responds elastically to changes in the level of the public good. Such elastic response exacerbates the free-riding problem at high match ratios and may sometimes result in a very small matching gift and a lower fundraising amount compared to seed money. While this finding provides an alternative explanation for the desirability of seed money, it is limited to environments with a relatively small number of donors and an elastic marginal utility of the public good, which is violated in a wide range of commonly-used preferences (e.g. CES utility functions).[4] Instead, we focus on an alternative environment with many donors and standard inelastic preferences and find that the lower effectiveness of seed money is in fact an advantage under asymmetric information as it allows seed money to emerge as a costly signal of quality and consequently raise more funds compared to matching.

There is sparse theoretical literature that has considered incomplete information about the public good. Bag and Roy (2011) show that when donors have independent private valuations for the public good, free-riding incentives could diminish with sequential giving and thus sequential contributions might result in higher total donations compared to simultaneous ones. Krasteva and Yildirim (2013) consider an independent value threshold public good, in which each donor can choose whether to contribute informed or uninformed. They find that announcing seed money discourages informed giving while a matching gift encourages it. However, in both studies, the independence of donors' valuations precludes the possibility of signaling via the scheme choice or the contribution amount by the lead donor. In this respect, the closest papers to ours are Vesterlund (2003) and Andreoni (2006).

Similar to our model, Vesterlund (2003) and Andreoni (2006) consider the use of seed money as a signaling device to convey the charity's quality. They demonstrate that leadership gifts in the form of seed money may result in larger total donations compared to simultaneous contributions since seed money enables the lead donor to signal the charity's quality to

[4] In Section 3, we show that as the donor population grows, matching must eventually (weakly) outperform seed money irrespective of the elasticity level. Thus, the preference for seed money is limited to an environment with a small donor pool. Moreover, as Gong and Grundy (2014) illustrate, elastic marginal utility of the public good is a necessary, but not sufficient condition for seed money to result in higher overall donations than a matching gift. Thus, even with an elastic utility and a small donor pool, matching may still emerge as the dominant scheme.

subsequent donors. However, an important distinction between these papers and ours is that they only allow for a seed money leadership scheme and ignore the possible signaling value of a matching gift. By enabling charities to choose between seed money and matching, we allow them to use the structure of the leadership gift itself to convey quality information to donors. In particular, such quality signaling through the scheme becomes an important tool of information transmission when acquiring information about the charity's quality is costly for donors.

In the realm of experimental studies, Silverman et al. (1984), Frey and Meier (2004), Soetevent (2005), Croson and Shang (2008), and Shang and Croson (2009) find that donors respond positively to information about other donors' gift, and Güth et al. (2007) show the positive impact of leadership gifts in particular. Furthermore, field experiments by List and Lucking-Reiley (2002), and Landry et al. (2006) demonstrate that both the likelihood and the size of donations significantly increase with the seed money amount. More interestingly, Potters et al. (2005) find that when some donors are informed and others are not, sequential contributions are likely to emerge endogenously, with more informed donors choosing to contribute first. All of these findings support the theory of seed money having signaling value. Potters et al. (2007) confirm this in an experiment that compares sequential contributions with an informed lead donor to simultaneous contributions.[5]

The impact of matching gifts has also been studied experimentally. Eckel and Grossman (2003), Meier and Frey (2004), Eckel and Grossman (2006a), Eckel and Grossman (2006b), Eckel et al. (2007), Falk (2007), Eckel and Grossman (2008) find evidence in support of matching gifts being effective in boosting donations. However, Meier (2007) illustrates that this effect is short-lived and reverses in the long run. Moreover, Karlan and List (2007) find that while downstream donors respond positively to an announcement of a matching gift, they are unresponsive to an increase in the match ratio. More recently, Karlan et al. (2011) find little response to a matching gift and Adena and Huck (2017) find a negative response by donors.

All of the above studies focus on seed money or matching in isolation, but some of the recent literature directly compares the two schemes in the field. For example, Alpizar et al. (2008), Rondeau and List (2008), Huck and Rasul (2011), and Huck et al. (2015) find in a variety of field settings that knowledge of others' lump sum contribution amounts increases individual donations, but a reduction in the price of giving via a match (or reciprocal gift) has little impact. Moreover, Rondeau and List (2008) compare the effectiveness of both in the field and in a subsequent threshold public good lab experiment with complete information. They find that seed money is more effective in the field relative to the lab and conjecture that this

[5] Other related empirical work (e.g., Khanna and Sandler, 2000; Okten and Weisbrod, 2000; Andreoni and Payne, 2003; Andreoni and Payne, 2011) study the impact of government grants on private contributions. They find mixed results, which could be attributed to the differential impact of government grants on the fundraising effort by charities.

is due to the signaling value of the leadership gift in the field where donors' knowledge of the public good is likely limited. In contrast, the returns from the public good are known in the lab and thus the leadership gift conveys no signaling benefits. Our analysis confirms this intuition and provides a theoretical foundation for the above experimental findings.

In the following sections, we present our model and findings. Section 2 describes the theoretical model. Section 3 considers the benchmark case of complete information and describes how the fundraising schemes rank in terms of total contributions. Section 4 presents the full model with information asymmetry and endogenous information acquisition, and discusses the possibility of signaling through the fundraising scheme. Section 5 presents a few extensions to the base model and Section 6 concludes.

2 Model description

A single charity, C, aims to maximize the amount of money raised, G, to a continuous public good. The quality of the public good q takes two values- $q \in \{q_l, q_h\}$ with $0 < q_l < q_h$. The prior quality distribution is denoted by $\pi = \{\pi_l, \pi_h\}$ where $\pi_h \in (0, 1)$ stands for the probability of high quality.

On the contributors' side, the economy consists of a large set of donors, $\mathcal{D}$. Donors are characterized by their wealth $\in \{w_1, w_2, ..., w_I\}$ with $w_i > w_{i+1}$ for all $i \in [1, I-1]$. Moreover, $t_i \in (0, \infty)$ denotes the number of donors of wealth w_i. A donor of wealth w_i has the following preference over private and public consumption:

$$u_i(g_i, G, q) = h(w_i - g_i) + qv(G) \tag{1}$$

where $h'(\cdot) > 0, h''(\cdot) < 0, v'(\cdot) > 0, v''(\cdot) < 0$. Moreover, we assume that $qv'(0) > h'(w_1)$ and $\lim_{G\to\infty} qv'(G) = 0$, which jointly ensure positive and finite provision of the public good for all quality realizations.

The charity fundraises by employing leadership giving, in which it first solicits a lead donor, denoted by L, followed by simultaneous solicitation of all remaining donors, denoted by F. We let $w_L = w_1$ so that the lead donor belongs to the richest individuals in the economy. This is consistent with Andreoni (2006) who finds that the wealthiest individuals have the strongest incentives to become leaders in charitable campaigns. The leadership gift scheme, Z, chosen by C, can take the form of either seed money, S, or matching gift, M. Under S, L makes an unconditional contribution g_L^S that is publicly announced prior to the follower donors' contribution decisions. Under M, L commits to a match ratio m, which is publicly announced, and results in a contribution $g_L^M = m \sum_{i \in F} g_i^M = mG_F^M$ by L. To simplify the exposition, we denote the lead donor's choice by d_L^Z where $d_L^S = g_L^S$ and $d_L^M = m$.

The timing of the game is as follows. First, C privately observes q and publicly commits to a fundraising scheme $Z \in \{S, M\}$. Then, it solicits L for a donation. L privately decides whether to learn q at cost k and then publicly makes her contribution decision d_L^Z. All follower donors then observe Z and d_L^Z, and simultaneously choose their individual donations g_i^Z for $i \in F$.

The following sections provide the equilibrium characterization of the above game. We focus on a large economy, but our findings extend to any size economy, in which matching raises more donations than seed money for a given quality of the public good.[6] In particular, Section 3 presents a benchmark with observable quality and shows that a sufficiently large economy guarantees that matching always (weakly) dominates seed money. This is a foundational result that informs our analysis of unobservable quality, presented in Section 4.

3 Seed and matching in a large economy with observable quality

Given publicly observable quality q and fundraising scheme Z, each follower donor chooses her donation to maximize her payoff given by eq. (1). A contributing donor equates the marginal cost and benefit of donating, resulting in

$$h'(w_i - g_i^Z) = qv'(G^Z)\,(1 + m\mathbb{1}_M) \tag{2}$$

where $\mathbb{1}_M = 1$ for $Z = M$ and 0 otherwise. It is evident from eq. (2) that matching increases the marginal benefit of giving relative to seed money. Thus, for the same amount of total giving, i.e. $G^M = G^S$, i contributes more under the matching scheme as long as $m > 0$.

The concavity of $h(\cdot)$ and $v(\cdot)$ implies that higher anticipated G^Z reduces incentives to give and ensures the uniqueness of the equilibrium contributions by the follower donors.[7] Moreover, the decrease in the marginal value of the public good as a response to higher G^Z implies that each wealth "type" w_i has a drop-out level $G_i^{Z,0}(q, m\mathbb{1}_M)$ of the public good above which i becomes a non-contributor. $G_i^{Z,0}(q, m\mathbb{1}_M)$ solves

$$qv'(G_i^{Z,0})\,(1 + m\mathbb{1}_M) - h'(w_i) = 0 \tag{3}$$

Note that $G_i^{M,0}(q, m) > G_i^{S,0}(q)$ for any $m > 0$ as long as these drop-out levels are finite. The condition $\lim_{G\to\infty} qv'(G) = 0$ ensures that this is indeed the case for both schemes and all quality levels.[8] Moreover, $G_i^{Z,0}(q, m\mathbb{1}_M)$ is strictly decreasing in i for both schemes, which

[6] Our focus on a large economy is consistent with the size of the charitable giving market in the USA, in which 72% of contributions come from individual donations (see `https://www.nptrust.org/philanthropic-resources/charitable-giving-statistics/`).

[7] See Bergstrom et al. (1986) for proof of equilibrium existence and uniqueness.

[8] Yildirim (2014) provides a weaker condition for a finite drop-out level under seed money of $\frac{d}{dg_i} u_i(0, G, q) \leq 0$ for some G. Instead, our condition ensures that provision is finite under both seed money and matching. We discuss the possibility of infinite drop-out levels in Section 5.3 in the presence of warm-glow motivations for giving.

allows us to employ the Andreoni-McGuire algorithm (see Andreoni and McGuire, 1993) to derive the aggregate best response of the follower donors, $G_F^Z(q, d_L^Z)$, to a leadership gift d_L^Z. In particular, consistent with Yildirim (2014), let C^Z denote the set of contributing donors and G_F^Z-their total giving. Then if $C_i^Z = \{1, 2, ..., i\} \subseteq C^Z$, the short-fall in provision by C_i^Z is

$$\Delta_i^Z(q, G_F^Z, d_L^Z) = G_F^Z - \sum_{j=1}^{i} t_j \left[w_j - \phi\left(qv'(G_F^Z + g_L^Z)(1 + m\mathbb{1}_M) \right) \right] \tag{4}$$

where by definition $g_L^M = mG_F^M$ and $\phi(\cdot) = [h']^{-1}(\cdot)$ is strictly decreasing in its argument. Therefore, $\Delta_i^Z(q, G_F^Z, d_L^Z)$ is strictly increasing in G_F^Z. Thus, the Andreoni-McGuire algorithm uniquely pins down the set of contributing donors and their equilibrium donation amount $G_F^Z(q, d_L^Z)$. The following lemma extends the equilibrium characterization by Yildirim (2014) to include the possibility of matching[9].

Lemma 1 *Given d_L^Z, let $\Delta_i^{Z,0}(q, d_L^Z) = \Delta_i^Z(q, G_i^{Z,0}(q, m\mathbb{1}_M) - g_L^Z, d_L^Z)$ and C^Z denote the equilibrium set of contributors. Then, $i \in C^Z$ if and only if $\Delta_i^{Z,0}(q, d_L^Z) > 0$ and $i \in C^Z$ implies that $j \in C^Z$ for all $j < i$. Moreover, given $C^Z = \{1, 2, ..., e\}$, $G_F^Z(q, d_L^Z)$ uniquely solves $\Delta_e^Z(q, G_F^Z, d_L^Z) = 0$.*

Lemma 1 reveals that donor i becomes a contributor only if the follower donors with higher wealth than i fall short of providing the necessary contribution (i.e. $G_i^{Z,0}(q, m\mathbb{1}_M) - g_L^Z$) to reach i's drop-out level, $G_i^{Z,0}(q, m\mathbb{1}_M)$. Moreover, the contribution incentives are decreasing in i. Thus, deriving the equilibrium contributor set, C^Z, boils down to finding the highest i with $\Delta_i^0(q, d_L^Z) > 0$. Given C^Z, the equilibrium condition $\Delta_e(q, G_F^Z, d_L^Z) = 0$ requires that $G_F^Z(q, d_L^Z)$ eliminates any short-fall in contributions among C^Z, which precludes profitable deviation to higher giving by any of the contributing donors.

From eq. (4), it is evident that the size of the leadership gift has an impact on the follower donors' contributions. In particular, we focus on the effect of d_L^Z in a large economy, in which the set of contributing donors grows infinitely large. In particular, we let $\mathcal{D}_n$ denote the n-replica economy, in which the donor population of each wealth type w_i is replicated n times. Then, the following Proposition describes the equilibrium response to the leadership gift g_L^Z as n approaches infinity.

Proposition 1 *Let $\epsilon_v(G) = \frac{-v''(G)G}{v'(G)}$. Then, the follower donors' equilibrium response, $G_F^Z(q, d_L^Z)$, to d_L^Z for $Z = \{S, M\}$ is as follows:*

a) $G_F^S(q, g_L^S)$ is strictly decreasing in g_L^S whenever the contributor's set is non-empty ($C^S \neq \emptyset$), while the total contributions $G^{S,L}(q, g_L^S) = G_F^S(q, g_L^S) + g_L^S$ are strictly increasing in g_L^S. Moreover, $\lim_{n\to\infty} G^{S,L}(q, g_L^S) = G_1^{S,0}(q)$ with $\frac{dG_1^{S,0}(q)}{dg_L^S} = 0$.

[9] The proof of Lemma 1 is analogous to the proof of Proposition 2 in Yildirim (2014) and thus omitted here.

b) $G_F^M(q,m)$ *is strictly increasing in* m *if and only if* $\epsilon_v(G^{M,L}) < 1$ *where* $G^{M,L}(q,m) = (1+m)G_F^M(q,m)$ *denotes the total contributions. Moreover,* $G^{M,L}(q,m)$ *is strictly increasing in* m *and* $\lim_{n\to\infty} G^{M,L}(q,m) = G_1^{M,0}(q,m)$*, with* $G_1^{M,0}(q,0) = G_1^{S,0}(q)$ *and* $\frac{dG_1^{M,0}(q,m)}{dm} > 0$*.*

Part a) of Proposition 1 highlights the free-riding incentives present under seed money. As pointed out by Andreoni (1988) and more recently by Yildirim (2014), these incentives are exacerbated in the limit economy with seed money converging to the highest drop-out level $G_1^{S,0}(q)$. Since $G_1^{S,0}(q)$ is independent of the size of the leadership gift g_L^S, seed money is ineffective at increasing total contributions in a large economy. In contrast, part b) reveals that the follower donors' response to increasing m is positive as long as the marginal value of the public good is inelastic to an increase in total contributions $G^{M,L}$, i.e. $\epsilon_v(G^{M,L}) < 1$. Intuitively, an increase in m has two opposing effects. On the positive side, m reduces the effective price of giving and as a result increases the follower donors' marginal willingness to contribute. On the negative side, higher m also increases $G^{M,L}$ for a fixed giving by the follower donors, G_F^M. This, in turn, reduces the marginal willingness to contribute due to the free-riding incentives. The inelastic response to increasing $G^{M,L}$ ensures that the positive effect dominates the negative.[10] However, irrespective of which effect dominates, the overall impact of higher m is an increase in total giving $G^{M,L}(q,m)$. Moreover, in contrast to seed money, $G^{M,L}(q,m)$ is responsive to higher match ratios even in the limit economy. This is because, as revealed by eq. (3), matching increases the individual drop-out levels, making donors more willing to become contributors. As a result, matching converges to strictly higher total contributions compared to seed money for any non-zero match ratio.

Turning to the lead donor's problem, d_L^Z is chosen to maximize

$$u_L(q,d_L^Z) = h\left(w_1 - G^{Z,L}(q,d_L^Z) + G_F^Z(q,d_L^Z)\right) + qv\left(G^{Z,L}(q,d_L^Z)\right). \tag{5}$$

Differentiating $u_L(q,d_L^Z)$ with respect to d_L^Z gives rise to the following marginal utility of giving:

$$\begin{aligned} \frac{du_L(q,d_L^Z)}{dd_L^Z} &= \left[-h'\left(w_1 - G^{Z,L}(q,d_L^Z) + G_F^Z(q,d_L^Z)\right) + qv'(G^{Z,L}(q,d_L^Z))\right]\frac{dG^{Z,L}(q,d_L^Z)}{dd_L^Z} + \\ &+ h'\left(w_1 - G^{Z,L}(q,d_L^Z) + G_F^Z(q,d_L^Z)\right)\frac{dG_F^Z(q,d_L^Z)}{dd_L^Z} \end{aligned} \tag{6}$$

[10] Gong and Grundy (2014) show that if $\epsilon_v(G) > 1$ for some G, $G_F^M(q,m)$ may have an inverted U shape with donors reducing their donation amounts as a response to high match ratios. Consequently, the lead donor may settle for a low match. Then, in a finite economy, it is possible for matching to induce significantly lower leadership gift than seed money, resulting in lower overall donations under matching. This possibility, however, disappears in a large economy since, as stated by Propositions 1 and 2, seed money completely crowds out giving by the follower donors in the limit economy and thus is always less effective than matching in increasing public good provision.

Eq. (6) reveals that the follower donors' response to the leadership gift plays a crucial role in the lead donor's contribution choice. In particular, in a large economy, Proposition 1a) states that $\frac{dG^{S,L}(q,g_L^S)}{dg_L^S} \to 0$ as $n \to \infty$. Thus, the sign of $\frac{dG_F^S(q,g_L^S)}{dg_L^S}$ is the sole determinant of the lead donor's optimal contribution choice. Since Proposition 1a) reveals that $\frac{dG_F^S(q,g_L^S)}{dg_L^S} < 0$, it follows that $\lim_{n\to\infty} \frac{du_L(q,g_L^S)}{dg_L^S} < 0$ for all g_L^S. Thus, in a large economy, the lead donor has no incentives to contribute to the public good under seed money , i.e. $\lim_{n\to\infty} g_L^{S,*}(q) = 0$. This stands in contrast to the matching scheme, in which the lead donor's gift may encourage more giving by the follower donors. To see how this impacts the lead donor's willingness to give in a large economy, recall that the total contribution amount under M converges to $G_1^{M,0}(q,m)$. Moreover, since $G_1^{M,0}(q,0) = G_1^{S,0}(q)$ (by Proposition 1b)), for $m = 0$, eq. (6) reduces to

$$\lim_{n\to\infty} \frac{du_L(q,0)}{dm} = \left[-h'(w_1) + qv'(G_1^{S,0}(q))\right] \frac{dG_1^{M,0}(q,0)}{dm} + h'(w_1) \lim_{n\to\infty} \frac{dG_F^M(q,0)}{dm} \tag{7}$$

By eq. (3), the first term drops out, and $\lim_{n\to\infty} \frac{du_L(q,0)}{dm} = h'(w_1) \lim_{n\to\infty} \frac{dG_F^M(q,0)}{dm}$. Thus, the lead donor will always find it optimal to offer a positive match in a large economy as long as this induces higher giving by the follower donors. The following Proposition formalizes this finding.

Proposition 2 *In the limit economy ($n \to \infty$), the equilibrium total donations, $G_\infty^{Z,*}(q) = \lim_{n\to\infty} G^{Z,*}(q)$, satisfy $G_\infty^{M,*}(q) \geq G_\infty^{S,*}(q)$, with a strict inequality if $\epsilon_v(G_1^{S,0}(q)) < 1$.*

The sufficient condition provided by Proposition 2 implies that $\lim_{n\to\infty} \frac{dG_F^M(q,0)}{dm} > 0$. Thus, the matching scheme outperforms seed money due to its ability to reduce free-riding incentives by the follower donors. Together, Propositions 1 and 2 reveal that under complete information about the charity's quality, the fundraiser is likely to favor the matching scheme. Therefore, to understand the use of seed money, we next turn to an environment with incomplete information about the charity's quality.

4 Seed and matching in a large economy with unobservable quality

In this section, we extend our analysis to the incomplete information game described in Section 2. For the remainder of the analysis, we maintain the assumption of an inelastic marginal value of the public good, i.e. $\epsilon_v(G_1^{S,0}(q)) < 1$ for all q, which guarantees the strict preference of matching over seed money in a large economy by every charity type in the absence of information asymmetry. We show that this preference causes seed money to emerge as a costly signal of quality in the presence of limited information about q.

In the last stage of the game, the follower donors make simultaneous donation decisions corresponding to $G_F^Z(q_{\mathcal{F}}^Z, d_L^Z)$, derived in Section 3, where $q_{\mathcal{F}}^Z$ denotes the follower donors' belief about the charity's quality. Given the asymmetric access to information, the lead donor's utility is given by

$$\overline{u}_L(q_{\mathcal{L}}^Z, q_{\mathcal{F}}^Z, d_L^Z) = h\left(w_L - G^{Z,L}(q_{\mathcal{F}}^Z, d_L^Z) + G_F^Z(q_{\mathcal{F}}^Z, d_L^Z)\right) + q_{\mathcal{L}}^Z v\left(G^{Z,L}(q_{\mathcal{F}}^Z, d_L^Z)\right) \tag{8}$$

The above equation captures the possibility that the lead donor and the follower donors hold asymmetric beliefs about the charity's quality. In a sequential equilibrium, donors' beliefs have to be consistent with the charity's contribution strategy Z, and the lead donor's donation decision d_L^Z. In particular, letting $\beta^Z(q_j)$ denote the probability that a charity of type q_j for $j \in \{l, h\}$ chooses scheme Z, the posterior belief of type q_j upon observing scheme Z, denoted by π_j^Z, satisfies Bayes' rule:

$$\pi_j^Z = \frac{\beta^Z(q_j)\pi_j}{\sum_{y \in \{l,h\}} \beta^Z(q_y)\pi_y} \tag{9}$$

Given π_j^Z, the expectation of quality in absence of any additional information is simply the posterior expected value $q_U^Z = \sum_j \pi_j^Z q_j$. However, the lead donor may also choose to learn the charity's true quality at a cost k. Therefore, the lead donor's belief $q_{\mathcal{L}}^Z$ can take one of three possible values- $\{q_l, q_U^Z, q_h\}$, denoting the cases of informed low quality, uninformed, and informed high quality, respectively. From the viewpoint of the follower donors, $q_{\mathcal{L}}^Z$ is the lead donor's type. Letting α^Z denote the lead donor's likelihood of information acquisition in scheme Z, the probability of type $q_{\mathcal{L}}^Z$, denoted by $\eta_{\mathcal{L}}^Z$, satisfies

$$\eta_{\mathcal{L}}^Z = \begin{cases} \pi_{\mathcal{L}}^Z \alpha^Z & \text{for } \mathcal{L} \neq U \\ 1 - \alpha^Z & \text{for } \mathcal{L} = U \end{cases} \tag{10}$$

Note from eqs. (9) and (10) that whenever the two types of charities perfectly separate, such as $\beta^Z(q_l) = 1 - \beta^Z(q_h) = 1$, the scheme becomes perfectly informative with $q_U^Z = q_l$ and thus $q_{\mathcal{L}}^Z = q_l$ is independent of the information acquisition strategy of the lead donor. In this case, the lead donor's gift d_L^Z is not an essential tool of information transmission. In the spirit of sequential equilibrium, however, we consider equilibrium behavior that is consistent with the limit of fully mixed strategies. Therefore, the fundraising scheme choice always leaves some uncertainty about the charity's quality. As a result, the lead donor's gift d_L^Z has additional signaling value and can provide further information to the follower donors whenever the lead donor acquires information.

Turning to the choice of leadership gift d_L^Z, note that the lead donor's objective function, given by eq. (8), satisfies $\frac{\partial^2 \overline{u}_L(q_{\mathcal{L}}^Z, q_{\mathcal{F}}^Z, d_L^Z)}{\partial q_{\mathcal{L}}^Z \partial G^{Z,L}} = v'(G^{Z,L}) > 0$, which as shown in Lemma A-2 in the Appendix serves as the single crossing property that guarantees the existence of a separating equilibrium, in which the lead donor always reveals her type $q^Z_{\mathcal{L}}$ to the follower donors. In particular, we focus on the least costly (Riley) equilibrium, which is uniquely selected by the Cho-Kreps intuitive criterion (Cho and Kreps, 1987). The equilibrium contribution of the lead donor of type $q^Z_{\mathcal{L}}$ in the Riley equilibrium satisfies:[11]

$$\overline{d}_L^{Z,*}(q^Z_{\mathcal{L}}) = \underset{d^Z_L}{argmax} \quad \overline{u}_L(q^Z_{\mathcal{L}}, q^Z_{\mathcal{L}}, d^Z_L) \tag{11}$$

$$\text{s.t.} \quad \overline{u}_L(q_l, q_l, \overline{d}_L^{Z,*}(q_l)) \geq \overline{u}_L(q_l, q^Z_U, \overline{d}_L^{Z,*}(q^Z_U))$$
$$\overline{u}_L(q^Z_U, q^Z_U, \overline{d}_L^{Z,*}(q^Z_U)) \geq \overline{u}_L(q^Z_U, q_h, \overline{d}_L^{Z,*}(q_h))$$

Thus, each type $q^Z_{\mathcal{L}}$ chooses the contribution level that maximizes her utility subject to an incentive compatibility (IC) constraint ensuring that no lower quality type can profit from mimicking her contribution. As typical for the Riley solution, there is no distortion in the contribution level of the low quality type q_l. This, in turn, implies that the total contributions raised by a low quality charity coincides with the amount raised under complete information, i.e $\overline{G}^{Z,*}(q_l) = G^{Z,*}(q_l)$. Therefore, conditional on an informed lead donor, the low quality charity always raises more donations under matching. This comparison is less clear for the higher quality types, whose contribution amounts may be distorted towards higher contribution levels as a result of the IC constraints above. However, as pointed out by Andreoni (2006), in a large economy the equilibrium donations under seed money $\overline{G}^{S,*}(q_{\mathcal{L}})$ are bounded from above by the full information amount $G_1^{S,0}(q_{\mathcal{L}})$. By Lemma 1, any higher amount would turn all follower donors into non-contributors and thus cannot be sustained in equilibrium. This implies that in a large economy, the seed money contributions in the Riley equilibrium must necessarily fall below the matching contributions.[12] The following Lemma formalizes this statement.

Lemma 2 *For sufficiently large n, $\overline{G}^{M,*}(q_j) > \overline{G}^{S,*}(q_j)$ for $j \in \{l, h\}$ and $\overline{G}^{M,*}(q^M_U) > \overline{G}^{S,*}(q^S_U)$ for $q^M_U \geq q^S_U$.*

[11] Lemma A-2 shows that as typical in signaling games, the two constraints given by (11), ensuring no deviation incentives by the quality types q_l and q^Z_U of lead donor towards higher donation amounts, are the only ones that might bind in the least costly separating equilibrium.

[12] It is worth pointing out that with a relatively small donor base, it is possible for contributions under seed money in the Riley equilibrium to exceed the ones under matching for a fixed donor type q, i.e. $\overline{G}^{S,*}(q) > \overline{G}^{M,*}(q)$. Intuitively, with costly quality signaling, a low type of lead donor may be more reluctant to pool with a higher type under matching since the resulting higher donation amounts by the follower donors also increase the lead donor's contribution through the match. This can make separation by the high type of lead donor less costly under matching compared to seed money and as a result lead to lower overall contributions. While this finding is consistent with the anecdotal evidence alluded to in the Introduction, it is of limited scope and thus not the focus of our analysis.

Lemma 2 establishes that in a large economy matching necessarily dominates seed money whenever the lead donor is informed about the quality or whenever matching is associated with (weakly) higher quality level. The last observation follows from the fact that the expected equilibrium contributions are increasing in the donors' belief about the charity's quality.[13] Note that the expected total contributions in scheme Z by a charity of quality q_j depend both on the expected quality q_U^Z as well as the expected likelihood of information acquisition and are given by

$$\overline{G}_E^Z(q_j, q_U^Z, \alpha^Z) = \alpha^Z \overline{G}^{Z,*}(q_j) + (1-\alpha^Z)\overline{G}^{Z,*}(q_U^Z) \tag{12}$$

Given the above equation, Lemma 2 implies that in order for seed money to be attractive for the charity, it must either be associated with a higher expected quality than matching or result in more favorable information acquisition strategy by the lead donor.

In order to understand the lead donor's information acquisition incentives, we next turn to the lead donor's value of information. In particular, let $\overline{u}_L^{Z,*}(q_{\mathcal{L}}^Z) = \overline{u}_L(q_{\mathcal{L}}^Z, q_{\mathcal{L}}^Z, \overline{d}_L^{Z,*}(q_{\mathcal{L}}^Z))$ denote the optimal utility of type $q_{\mathcal{L}}^Z$ from the contribution stage of the game. Anticipating this utility level, the value of informed giving for the lead donor is simply the difference between the expected informed and uninformed utility:

$$V_I^Z(\pi^Z) = \pi_h^Z \overline{u}_L^{Z,*}(q_h) + \pi_l^Z \overline{u}_L^{Z,*}(q_l) - \overline{u}_L^{Z,*}(q_U^Z) \tag{13}$$

The value of information depends crucially on the charity's equilibrium fundraising strategy through its effect on q_U^Z and π^Z. In particular, the following lemma points out that $V_I^Z(\pi^Z)$ is positive if and only if the two charity types (partially) pool in equilibrium, thus leaving the lead donor uncertain of the charity's quality.

Lemma 3 *$V_I^Z(\pi^Z)$ is continuous in $\pi_h^Z \in [0,1]$. Moreover, $V_I^Z(\pi^Z) = 0$ for $\pi_h^Z \in \{0,1\}$ and $V_I^Z(\pi^Z) > 0$ for all $\pi_h^Z \in (0,1)$.*

The value of information is always non-negative as more informed giving allows the lead donor to better tailor her contribution to the value of the public good. In the extreme case of the two charity types following a fully separating fundraising strategy, the fundraising scheme is perfectly informative, i.e. $\pi_j^Z = 1$ for some $j \in \{l, h\}$, rendering information acquisition inconsequential. Such fully separating equilibrium, however, is rather incidental as it holds for very limited set of parameter values. Note that in such an equilibrium, it must be the case that the low quality charity chooses the matching scheme, while the high quality charity chooses seed money. Otherwise, if matching is a pure signal of high quality, by Lemma 2, $\overline{G}^{M,*}(q_h) > \overline{G}^{S,*}(q_l)$ and thus the low quality charity will for sure have incentives to mimic the high type and deviate to matching. Therefore, in a fully separating equilibrium, it must

[13] See Claim 1 in the proof of Lemma A-2.

be the case that seed money is a pure signal of high quality. Moreover, in order to prevent deviation by either type of charity, the two schemes must raise the same amount of money ($\overline{G}^{S,*}(q_h) = \overline{G}^{M,*}(q_l)$). This makes the fully separating equilibrium rather incidental. However, the observation that both charity types must generate the same amount of equilibrium donations extends to any equilibrium with no information acquisition, as highlighted by the following Proposition.

Proposition 3 *(Fully uninformed equilibria) In every equilibrium with no information acquisition on the equilibrium path, i.e., $\alpha^{Z,*} = 0$ for all Z with $\sum_j \beta^{Z,*}(q_j) > 0$, each scheme on the equilibrium path results in the same total donations and each charity raises the same amount of money.*

In the absence of information acquisition, the high quality charity is not able to effectively separate from the low quality charity. This is because the charity's payoff function does not satisfy the single crossing property and thus imitation by the low type is completely costless in this case. Consequently, the two charities will either pool on the same scheme or the two schemes would be equally attractive to prevent profitable deviation. Since this is not consistent with the experimental evidence alluded to in the Introduction, we instead focus on equilibria in which information acquisition occurs with positive probability.

In order for information acquisition to take place, the value of information should be sufficiently high relative to the cost. In particular, if the value of information under matching at the prior distribution $V_I^M(\pi)$ exceeds the cost k, the other extreme case of fully informed equilibrium always exists. Such fully informed equilibrium, however, requires both charity types to pool on the matching scheme, as revealed by the following Proposition.

Proposition 4 *(A fully informed equilibrium) Fully informed equilibrium with $\alpha^{Z,*} = 1$ for all Z on the equilibrium path (i.e. $\sum_j \beta^{Z,*}(q_j) > 0$) exists if and only if $V_I^M(\pi) \geq k$. Moreover, the fully informed equilibrium is unique with $\beta^{M,*}(q_j) = 1$ for all $j \in \{l, h\}$, $\alpha^{M,*} = 1$, and $\overline{G}^{M,*}(q_h) \geq G^{M,*}(q_h)$.*

Proposition 4 is an immediate consequence of Lemma 2 that reveals the superiority of matching over seed money for a fixed quality level. Intuitively, as long as the lead donor obtains information with certainty, the high quality charity can fully rely on the lead donor to signal this quality to the follower donors through her donation choice. As a result, matching is preferred by both charity types since it incentivizes more giving. Interestingly, the amount of money raised by the high quality charity in equilibrium exceeds the amount raised under complete information ($\overline{G}^{M,*}(q_h) \geq G^{M,*}(q_h)$). This is because the lead donor's contribution is tailored to signal away from the low quality type, which may require a match that exceeds the one chosen under complete information. In this respect, limited quality transparency on the market can in fact benefit the high quality charity by increasing the lead donor's contribution amount.

Proposition 4 implies that the lead donor must have reduced incentives to acquire information in order for the high quality charity to find seed money attractive. However, Proposition 3 indicates that the other extreme of no information acquisition also does not provide strict incentives for seed money fundraising. Thus, we next turn to partial information acquisition. In particular, we focus on equilibria with partial information acquisition, in which seed money is on the equilibrium path[14]. We refer to such equilibria as *SPI* (seed-partial info) equilibria. More formally, the likelihood of scheme Z emerging in equilibrium, $E[\beta^{Z,*}]$, and the corresponding expected likelihood of information acquisition, $E[\alpha^*]$, are given by

$$E[\beta^{Z,*}] = \pi_h \beta^{Z,*}(q_h) + (1 - \pi_h)\beta^{Z,*}(q_l) \tag{14}$$

$$E[\alpha^*] = \sum_{Z \in \{S,M\}} E[\beta^{Z,*}]\alpha^{Z,*} \tag{15}$$

The following statement provides a formal definition of a *SPI* equilibrium.

Definition 1 *SPI equilibrium satisfies* $E[\beta^{S,*}] > 0$ *and* $E[\alpha^*] \in (0, 1)$.

A *SPI* equilibrium requires both that seed money is chosen with positive probability by some quality type and that there is limited information acquisition on the equilibrium path. Note that limited information may arise as a result of randomization in the information acquisition strategy by the lead donor for a given scheme or the lead donor's asymmetric information acquisition strategy under the two schemes. The following Lemma provides sufficient conditions for the existence of a *SPI* equilibrium and some notable properties.

Lemma 4 *(Existence of a SPI equilibrium) A SPI equilibrium exists if* $V_I^S(\pi) \geq k$ *and* $\overline{G}^{S,*}(E[q]) > \overline{G}^{M,*}(q_l)$. *Moreover, every SPI equilibrium satisfies 1)* $\beta^{S,*}(q_j) > 0$ *for all* $j \in \{l, h\}$*; 2)* $\alpha^{M,*} < 1$ *and* $\alpha^{S,*} \in (0, 1)$.

Lemma 4 states that a *SPI* equilibrium exists as long as the cost of information is low relative to the value of information under seed money at the prior π (i.e, $V_I^S(\pi) \geq k$), and the prior expected quality is high enough so that the uninformed seed money fundraising at the prior is sufficiently attractive for the low type (i.e, $\overline{G}^S(E[q]) > \overline{G}^M(q_l)$). This is because, as stated by the first property, both charity types must be present in seed money. To understand the first property, note that the low type would never unilaterally choose seed money since

[14] As typical for most signaling games, there is multiplicity of equilibria, including an equilibrium, in which seed money is off the equilibrium path due to very pessimistic beliefs about the charity's quality. For our purposes, however, the more relevant equilibria involve seed money being utilized by charities in equilibrium since it allows us to address the question of which type of charity is more likely to employ seed money fundraising.

it would perfectly reveal its quality. The high type, on the other hand, may find seed money attractive if it is perfectly revealing of its quality, but the resulting zero value of information and no quality verification by the lead donor, would make seed money also attractive for the low type. Thus, in equilibrium, both types need to utilize seed money, resulting in strictly positive value of information (Lemma 3).

Given the presence of both types in seed money, the second property in Lemma 4 requires that information acquisition is less than perfect under the matching scheme and that the lead donor strictly randomizes in her information acquisition strategy under seed money. Less than perfect information acquisition under matching ($\alpha^{M,*} < 1$) and some information acquisition under seed money ($\alpha^{S,*} > 0$) is necessary in order for the high type to consider seed money fundraising. In addition, limited information acquisition under seed money $\alpha^{S,*} < 1$ is required in order to make seed money attractive for the low type.

Lemma 4 establishes that with partial information acquisition, seed money cannot be a perfectly revealing signal of quality. Nevertheless, we are interested in how seed money compares to matching in conveying quality information to donors. The following Proposition delivers a sharp prediction by establishing that in any *SPI* equilibrium, seed money is a stronger signal of high quality compared to matching.

Proposition 5 *In every SPI equilibrium, the seed money scheme is associated with higher expected quality, i.e.* $q_U^{S,*} > q_U^{M,*}$*, and higher expected donations, i.e.* $E_j\left[\overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*})\right] > E_j\left[\overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*})\right]$*, for* $j \in \{l, h\}$.

Proposition 5 is consistent with the experimental evidence alluded to in the Introduction. It reveals that in every *SPI* equilibrium, seed money is associated with higher expectation of quality, which implies that it is chosen by the high quality charity more frequently that by the low quality charity. Intuitively, the attraction of seed money for the high quality charity is in its ability to signal the charity's quality more reliability. Thus, by eq. (12), seed money must be either associated with higher expected quality for the uninformed lead donor or induce more information acquisition by the lead donor relative to matching. However, if the benefit is coming purely from more information acquisition, such that $\alpha^{S,*} > \alpha^{M,*}$ and $q_U^{M,*} > q_U^{S,*}$, then the low quality charity would strictly prefer to fundraise for matching. This is because unlike the high type, the low type dislikes information acquisition and would find matching more attractive if it is less informative and associated with more optimistic belief regarding its type. Thus, a necessary condition for both types to find seed money attractive is for seed money to signal higher quality to donors.

An immediate consequence of the higher posterior belief under seed money (i.e. $\pi_h^{S,*} > \pi_h^{M,*}$) is that seed money raises higher expected donations relative to matching. To see this, note that since both charity types choose seed money with positive probability (by Lemma 4), it must be true that seed money generates at least as much expected contributions as matching for either type, i.e. $\overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*}) \geq \overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*})$. However, since these expected contributions are strictly increasing in the charity's quality, high contributions are more frequent under seed money relative to matching. As a result, the overall expected donations are higher under seed money.

In terms of the fundraising strategies, the *SPI* equilibrium is not unique. While both types need to be present in seed money (Lemma 4), this is not necessarily the case for the matching scheme. The possible equilibrium strategies vary with both types pooling on seed money, only the low type being present in matching, or each type being present in both schemes. The more interesting equilibria involve both schemes being on the equilibrium path. Thus, in the remainder of this section, we focus on characterizing this set of *SPI* equilibria.

For any equilibrium with strict mixing in information acquisition under Z, it must be the case that the value of information is equal to the cost. Let $(\hat{\pi}^S, \hat{\pi}^M)$ denote the pair of posterior beliefs that satisfy the following conditions:

Definition 2 *The set of posterior beliefs $(\hat{\pi}^S, \hat{\pi}^M)$ with corresponding expected qualities $(\hat{q}_U^S, \hat{q}_U^M)$ satisfy:*

$$
\begin{aligned}
C2: &\quad V_I^Z(\hat{\pi}^Z) = k \quad \text{for} \quad Z = \{S, M\} \\
C3: &\quad \hat{\pi}_h^S > \hat{\pi}_h^M
\end{aligned}
$$

In the Appendix, we show that as long as the value of information under the prior exceeds the cost for each scheme, i.e. $V_I^Z(\pi) \geq k$, there always exits a (non)degenerate strategy by the two types of charities that guarantees a pair of posterior beliefs that satisfy C2 and C3. Using this property, the following Proposition describes the equilibrium strategies by the two charities that emerge under a *SPI* equilibrium.

Proposition 6 *Consider SPI equilibria, in which M is on the equilibrium path.*

1) *If $V_I^S(\pi) \geq k$ and $\overline{G}^{S,*}(E[q]) > \overline{G}^{M,*}(q_l)$, there exists an equilibrium with $\beta^{S,*}(q_h) = 1$ and $\beta^{S,*}(q_l) \in (0,1)$ satisfying $V_I^S\left(\pi^{S,*}\right) = k$.*

2) *If $V_I^Z(\pi) > k$ for all Z and $\overline{G}^{S,*}(\hat{q}_U^S) > \overline{G}^{M,*}(\hat{q}_U^M)$, there exists a fully non-degenerate equilibrium with*
$$\beta^{S,*}(q_h) = \frac{\hat{\pi}_h^S}{\pi_h}\frac{\pi_h - \hat{\pi}_h^M}{\hat{\pi}_h^S - \hat{\pi}_h^M}; \beta^{S,*}(q_l) = \frac{1-\hat{\pi}_h^S}{1-\pi_h}\frac{\pi_h - \hat{\pi}_h^M}{\hat{\pi}_h^S - \hat{\pi}_h^M} \tag{16}$$
where $0 < \beta^{S,*}(q_l) < \beta^{S,*}(q_h) < 1$.

Proposition 6 characterizes two types of equilibria. In the first one, only the low quality type chooses matching, making matching a sure signal of low quality, while both types are present in seed money. Note that in such an equilibrium, the low type of charity is indifferent between the two schemes and in equilibrium randomizes to make the lead donor indifferent in her information acquisition strategy under seed money. To guarantee the existence of such an equilibrium, the cost of information should be sufficiently low to ensure some information acquisition under seed money in equilibrium. Moreover, the low quality charity should raise significant donations under seed money when the lead donor is uninformed to compensate for the lower donations when she is informed.

In the second equilibrium, both charities are randomizing between matching and seed money. This equilibrium is important since it illustrates that both schemes could be used by the two charity types. Thus, neither schemes is perfectly informative, but rather in equilibrium the follower donors use both the fundraising scheme and the size of the lead donor's gift to infer information about the charity's quality. This equilibrium requires not only that seed money is sufficiently lucrative for the low type when the lead donor is uninformed, but also that uninformed donations raised under matching are low enough to make seed money an attractive option for the high type.

Overall, the analysis in this section illustrates that with costly information acquisition, seed money is likely used by the high quality charity to credibly signal its quality. More importantly, we illustrate that with both schemes being utilized in equilibrium, the seed money scheme is always indicative of a higher expected quality compared to the matching scheme. This is a rather strong result that provides a feasible explanation for the recent experimental findings. In the next Section, we discuss a few extensions and variations of the model both to highlight the robustness of this finding and to inform how the signaling via the fundraising scheme is affected by factors such as the possibility of opting out of leadership fundraising, the presence of an alternative credible signal of quality, and warm-glow incentives for giving among donors.

5 Model extensions and variations

This section extends our model in multiple directions. Section 5.1 illustrates that the role of seed money as a signal of higher quality extends to an arbitrary finite quality distribution. Section 5.2 studies the impact of expanding the set of scheme choices by the charity to allow for no leadership giving, while Section 5.3 studies the impact of warm-glow motivations for giving on the signaling role of the fundraising scheme. Both extensions illustrate the robustness of our results to richer environments. Section 5.4 studies the impact of an alternative information source to donors. It establishes that the presence of such information decreases the lead donor's value of information under seed money, which in turn reduces the possibility of *SPI* equilibria.

5.1 Multiple quality types

Consider an extension of the base model to finite quality levels where $q \in \{q_1, q_2, ..., q_t\}$ with $t > 2$ and $q_{j-1} < q_j$ for all $j \in (2, t]$. The corresponding distribution of types $\pi = (\pi_1, \pi_2, ..., \pi_t)$ denotes the likelihood of each type prior to any action being taken by the players. The information structure and timing of the game is identical to the base model.

Analogous to the base model, the lead donor's type $q^Z_{\mathcal{L}} \in \{q_1, ...q_j, q^Z_U, q_{j+1}, ..., q_t\}$ can take $t+1$ values as it includes the possibility of the lead donor choosing to remain uninformed, where her type is the expected quality $q^Z_U = \sum_{j=1}^{t} \pi^Z_j q_j$. Given the probability of information acquisition, α^Z, and letting $\mathcal{L} = \{1, 2, ..., t\} \cup \{U\}$, the prior belief, $\eta^Z_{\mathcal{L}}$ is given by eq. (10).

Similar to the two type case, in the least costly separating equilibrium the lead donor's contribution amount is perfectly informative of her type with $\overline{G}^M(q_{\mathcal{L}}) > \overline{G}^S(q_{\mathcal{L}})$ for all $q_{\mathcal{L}}$ in a large economy. The corresponding value of information is

$$V^Z_I(\pi^Z) = \sum_{j=1}^{t} \pi^Z_j \overline{u}^{Z,*}_L(q_j) - \overline{u}^{Z,*}_L(q^Z_U) \tag{17}$$

It is straightforward to verify that the fully informed equilibrium exists as long as $V^M_I(\pi) \geq k$ and necessitates pooling on matching. The other extreme of fully uninformed equilibrium requires each charity and each scheme on the equilibrium path to raise the same amount of money. Thus, similar to the two-type case, we focus our analysis on *SPI* equilibria defined by Definition 1. The following Lemma states that in any *SPI* equilibrium, information acquisition has to be limited under matching and positive under seed money to induce seed money fundraising by some charity types.

Lemma 5 *Every SPI equilibrium satisfies 1)* $\alpha^{M,*} < 1$ *and* $\alpha^{S,*} > 0$*; 2)* $\pi^{S,*}_j < 1$ *for all* $j \in \{1, ..., t\}$.

Limited information acquisition under matching ($\alpha^{M,*} < 1$) is necessary to prevent unraveling, in which each charity type deviates to matching. To see this, note that with full information under matching, total donations under matching must exceed total donations under seed money for each charity with above average seed money quality, $q_{\mathcal{L}} > q^{S,*}_U$, since $\overline{G}^{M,*}(q_{\mathcal{L}}) > \overline{G}^{S,*}(q_{\mathcal{L}}) > \overline{G}^{S,*}(q^{S,*}_U)$. Intuitively, a charity is willing to solicit for seed money only if it generates more favorable beliefs about its type under seed money. This implies that any charity above the average quality $q^{S,*}_U$ would prefer to avoid seed money. This would reduce the expected quality under seed money, causing further unraveling, in which all charities gravitate towards matching. Thus, to induce seed money fundraising, matching should be associated with less than perfect information acquisition.

Similar dynamics as the one described above would take place if there is no information acquisition under seed money. Then, by the definition of a *SPI* equilibrium, $\alpha^{M,*} > 0$. Thus, the expected giving under matching $\overline{G}_E^{M,*}(q_{\mathcal{L}}, q_U^{M,*}, \alpha^{M,*})$ is strictly increasing in $q_{\mathcal{L}}$, while the expected giving under seed money, $\overline{G}^{S,*}(q_U^{S,*})$, is uniform across the charities. This implies that the highest quality types would choose matching and thus the expected quality under matching, $q_U^{M,*}$, should exceed the one under seed money, $q_U^{S,*}$. Consequently, any type $q_{\mathcal{L}} > q_U^{S,*}$ would have strict incentives to deviate to matching, further reducing $q_U^{S,*}$ and causing all charity types to gravitate towards matching. Thus, some information acquisition under seed money ($\alpha^{S,*} > 0$) is necessary to make seed money fundraising attractive.

The second property in Lemma 5 follows immediately from the first one. In order for information acquisition to take place under seed money, it must be the case that the value of information is positive, which necessitates (partial) pooling, i.e. $\pi_j^{S,*} < 1$. Even though seed money is only partially informative about the charity's quality in equilibrium, the following Proposition states that it is associated with higher expected quality relative to matching.

Proposition 7 *In every SPI equilibrium, the seed money scheme is associated with higher expected quality, i.e.,* $q_U^{S,*} > q_U^{M,*}$.

Proposition 7 generalizes our main result by showing that for arbitrary discrete distribution of types, seed money on the equilibrium path must be associated with higher expected quality relative to matching. To glean more insight into the equilibrium forces that drive this result, note that the highest type present in seed money $\overline{q}^S > q_U^{S,*}$ must necessarily exceed the expected quality under match, i.e. $\overline{q}^S > q_U^{M,*}$ to prevent $\overline{q}^S$ from deviating to matching[15]. Moreover, $\overline{q}^S$ finds seed money attractive either because it leads to higher uninformed giving, implying $q_U^{S,*} > q_U^{M,*}$, or has an informational advantage over matching, $\alpha^{S,*} > \alpha^{M,*}$. However, if the advantage is coming purely from information acquisition, then the lowest type under seed money $\underline{q}^S$ must have strict incentives to deviate to matching. To see this, note that by definition $\underline{q}^S < q_U^{S,*} < q_U^{M,*}$, implying that information acquisition is never good news for $\underline{q}^S$. Thus, matching would be a more attractive option for $\underline{q}^S$ as it is both less informative and associated with more optimistic beliefs about its type. This shows that $q_U^{S,*} > q_U^{M,*}$ is necessary to prevent deviation by both the highest ($\overline{q}^S$) and the lowest ($\underline{q}^S$) type under seed money.

Unlike the two-types case, characterizing the full set of *SPI* equilibria can be challenging. In the two-types model, the higher expected quality under seed requires that the high quality charity chooses seed money more often than the low quality charity. Thus, the likelihood of choosing seed money has to be monotonically increasing in quality. This monotonic relationship no longer needs to hold with multiple quality types as the following example illustrates.[16]

[15]Note that if $q_U^{M,*} > \overline{q}^S > q_U^{S,*}$, then $\overline{G}^{M,*}(q_U^{M,*}) > \overline{G}^{M,*}(\overline{q}^S) > \overline{G}^{S,*}(\overline{q}^S) > \overline{G}^{S,*}(q_U^{S,*})$.

[16]Proposition 7 establishes that the equilibrium posterior distribution under seed money $\pi^{S,*}$ second-order scholastically dominates the one under matching, $\pi^{M,*}$. With two types, second-order stochastic dominance

Example 1 *Let* $u_i(g_i, G, q) = (w_i - g_i)^{0.9} + qG^{0.1}$ *with* $w_1 = 2000$*, and* $q \in \{50, 500, 600, 700, 2000\}$ *with* $\pi = (0.65, 0.1, 0.05, 0.05, 0.15)$*. For* $k = 17.67$*, the following strategies constitute a sequential equilibrium in the limit economy:*

$$\beta^{S,*}(q) = (0, 1, 0, 0, 1),\ \alpha^{S,*} = 1,\ \alpha^{M,*} = 0.48$$

To verify this equilibrium, note that the posterior beliefs and expected qualities are:

$$\pi^{S,*} = (0, \frac{4}{10}, 0, 0, \frac{6}{10}) \quad , \quad \pi^{M,*} = (\frac{13}{15}, 0, \frac{1}{15}, \frac{1}{15}, 0) \quad , \quad q_U^{S,*} = 1400 \quad , \quad q_U^{M,*} = 130$$

To derive the equilibrium contributions, note that by Proposition 1 and eq. (3), $G^{S,L}(q_{\mathcal{F}}, d_L^S) = \left(\frac{.9w_1^{-.1}}{.1q_{\mathcal{F}}}\right)^{\frac{1}{-.9}}$ *and* $G^{M,L}(q_{\mathcal{F}}, d_L^M) = \left(\frac{.9w_1^{-.1}}{.1(m+1)q_{\mathcal{F}}}\right)^{\frac{1}{-.9}}$*. The lead donor's contribution for each type* $q_{\mathcal{L}}$*,* $\overline{d}_L^{Z,*}(q_{\mathcal{L}})$*, maximizes* $\overline{u}_L(q_{\mathcal{L}}, q_{\mathcal{L}}, d_L^Z)$ *subject to* $\overline{u}_L(q_{\mathcal{L}-1}, q_{\mathcal{L}-1}, \overline{d}_L^{Z,*}(q_{\mathcal{L}-1})) \geq \overline{u}_L(q_{\mathcal{L}-1}, q_{\mathcal{L}}, \overline{d}_L^{Z,*}(\mathcal{L}))$*, where* $q_{\mathcal{L}-1}$ *denotes the highest type below* $q_{\mathcal{L}}$*.*[17] *The numeric solution to this constrained optimization problem is*[18]*:*

q_j	$\overline{g}_L^{S,*}(q_j)$	$\overline{G}^{S,*}(q_j)$	$\overline{m}^*(q_j)$	$\overline{G}^{M,*}(q_j)$	$\overline{G}_E^{S,*}(q_j)$	$\overline{G}_E^{M,*}(q_j)$
50	0	15.64	0.10	17.39	15.64	46.61
500	45.55	202.01	0.56	331.09	202.01	197.19
600	86.76	247.37	0.70	446.07	247.37	252.38
700	129.26	293.58	0.77	553.68	293.58	304.03
2000	610.54	942.59	0.78	1788.82	942.59	896.90
$q_U^{S,*}$	361.23	634.18	–	–	–	–
$q_U^{M,*}$	–	–	0.55	73.59	–	–

where $\overline{G}^{Z,*}(q_j) = G^{Z,L}(q_{\mathcal{F}}, \overline{d}_L^{Z,*}(q_j))$ *and* $\overline{G}_E^{Z,*}(q_j)$ *is the equilibrium expected giving defined by eq. (12). Comparing* $\overline{G}_E^{S,*}(q_j)$ *and* $\overline{G}_E^{M,*}(q_j)$ *establishes* $\beta^{S,*}(q) = (0, 1, 0, 0, 1)$*. Moreover, by eq. (17)* $V_I^S(\pi^{S,*}) = 40.25 > V_I^M(\pi^{M,*}) = 17.67 = k$*. Thus, the lead donor has no incentive to deviate from* $\alpha^{S,*} = 1$ *and* $\alpha^{M,*} = 0.48$*.*

5.2 Opting out of leadership giving

So far, we have assumed that the charity always chooses to reveal the lead donor's gift and thus the only decision that the charity makes is whether to ask for seed or matching leadership

implies first-order stochastic dominance as well, which in turn requires a monotonically increasing relationship between the charity's quality and the likelihood of seed money. With multiple types, second-order stochastic dominance does not require such monotonic relationship.

[17] Lemma A-2 establishes that $\overline{u}_L(q_{\mathcal{L}-1}, q_{\mathcal{L}-1}, \overline{d}_L^{Z,*}(q_{\mathcal{L}-1})) \geq \overline{u}_L(q_{\mathcal{L}-1}, q_{\mathcal{L}}, \overline{d}_L^{Z,*}(\mathcal{L}))$ is the only constraint that might be binding at the optimum.

[18] Our numeric solution allows for lump sum donations and match ratios to be multiples of 0.01.

gift. One may wonder how the relative appeal of the two leadership schemes may change if we allow the charity to opt out of leadership giving completely. In the spirit of Vesterlund (2003), suppose that the charity can commit not to announce (N) the lead donor's contribution and instead to solicit each donor for an unconditional gift. This turns the contribution game into a simultaneous game, precluding the possibility of signaling by the lead donor and leaving the scheme choice as the only possible source of information.

It is important to note that unlike Vesterlund (2003), who allows the lead donor to donate multiple times, we model the lead donor's decision as a one-time contribution. However, this distinction becomes immaterial in a large economy. As pointed out by Vesterlund (2003), under symmetric quality information, sequential and simultaneous contributions raise the same amount of money if the lead donor is allowed to contribute multiple times. This equivalence also holds in the limit economy with purely altruistic donors. This is because, as pointed out by Proposition 1, the lead donors' seed money gift is completely crowded out in the limit, making seed leadership giving inconsequential under complete information. Thus, in the limit economy, the main distinction between seed money and non-announcement must come from the quality information conveyed to donors. Our analysis in this section focuses on this case.[19]

The equivalence of seed money and non-announcement under complete information implies that matching is still the dominant scheme. Under endogenous information acquisition, the comparison of the three schemes is less clear since the use of a leadership scheme does not guarantee informed contributions. Similar to the main model, in the absence of information acquisition, the two charity types must raise the same amount of money under any scheme on the equilibrium path since the lack of verification makes it costless for the low type to mimic the high type. Interestingly, however, fully informed equilibrium, in which the lead donor acquires information with probability one, no longer guarantees the use of matching. Recall from Section 4 that the low quality charity favors matching over seed money if her type is fully revealed in equilibrium. Non-announcement, however, provides means for the low quality charity to pool with the high even if the lead donor chooses to acquire information. The high quality charity may also favor non-announcement if matching is associated with sufficiently pessimistic beliefs about the charity's quality, forcing it off the equilibrium path. Nevertheless, a fully informed equilibrium precludes the possibility of seed money as stated by the following Lemma.

Lemma 6 *In the limit economy ($n \to \infty$), any fully informed equilibrium (i.e., $\alpha^{Z,*} = 1$ for all Z on the equilibrium path) requires that seed money is chosen with zero probability (i.e. $\beta^{S,*}(q_j) = 0$ for all $j = \{l, h\}$). Moreover, the two types of charity pool either on matching ($\beta^{M,*}(q_j) = 1$ for all*

[19] While for the sake of brevity we focus on the limit economy, similar to our base model, it can be shown that our main insights hold in the case of a finite, but large economy.

$j = \{l, h\}$) or non-announcement ($\beta^{N,*}(q_j) = 1$ for all $j = \{l, h\}$).

The intuition behind Proposition 6 is straightforward. Given an informed lead donor, matching always dominates seed money for the low charity type. Thus, seed money can be chosen only by the high type, which in turn results in no verification under seed money. Therefore, fully informed equilibrium precludes the use of seed money.[20]

The second part of Lemma 6 rules out the possibility of both non-announcement and matching being on the equilibrium path at the same time in a fully informed equilibrium. This is because, as shown in Lemma A-3 in the appendix, the contribution amount raised by the high quality charity in the limit economy under non-announcement, $\overline{G}_{\infty}^{N,*}(q_h, q_U^N, \alpha^N)$, is bounded from above by the corresponding seed money contributions, $\overline{G}_{\infty}^{S,*}(q_h)$, due to the lead donor's inability to signal the charity's quality under non-announcement. Moreover, from Proposition 2, we know that matching performs strictly better than seed money in a fully informed equilibrium for any quality type. It follows that the high quality charity would also strictly prefer matching over non-announcement if matching is on the equilibrium path and results in full verification. This, in turn, implies that non-announcement has to be associated with low quality, making it unattractive for the low quality charity as well. Clearly, both types choosing the matching scheme under full verification is sustainable with an off-equilibrium belief that non-announcement and seed money are associated with low quality. Both types choosing non-announcement with off-equilibrium belief of low quality under matching and seed is also sustainable as long as the low quality charity raises more funds by pooling with the high type under non-announcement than getting the low quality contributions under matching, i.e. $\overline{G}_{\infty}^{N,*}(q_l, q_U, 1) > \overline{G}_{\infty}^{M,*}(q_l)$.[21]

Similar to the main model, Lemma 6 implies that seed money should be associated with partial information acquisition in order to attract both charity types. The following Proposition establishes the possibility that seed money conveys the strongest signal of quality in equilibrium.

Proposition 8 *Every SPI equilibrium satisfies $q_U^{S,*} > q_U^{M,*}$. Moreover, if $V_I^S(\pi) \geq k$ and $\overline{G}_{\infty}^{S,*}(E[q]) > \overline{G}_{\infty}^{M,*}(q_l)$, there exists a SPI equilibrium, in which seed money is associated with the highest expected quality, $q_U^{S,*} > \max\{q_U^{M,*}, q_U^{N,*}\}$.*

The first part of Proposition 8 generalizes our main finding and establishes that the presence of non-announcement does not impact the relative quality comparison of seed money

[20] Lemma 6 stands in contrast to Vesterlund (2003), who shows the existence of an equilibrium, in which seed money results in full information acquisition by the lead donor. The possibility of matching and the fact that the low quality is non-zero ($q_l > 0$) precludes such equilibrium in our setting since the inability of the low quality charity to pool with the high under seed money makes matching strictly more attractive for the low quality charity.

[21] In Lemma A-3, we show that $\overline{G}_{\infty}^{N,*}(q_l, q_U^N, 1) > \overline{G}_{\infty}^{S,*}(q_l)$ for $q_U^N > q_l$ since non-announcement allows the low quality charity to conceal their quality from downstream donors. This makes it possible for the low quality charity to raise more funds under non-announcement relative to seed money in an incomplete information setting.

and matching gift. This is intuitive in light of the earlier discussion. The second part of the Proposition establishes the possibility that seed money is also a stronger quality signal relative to non-announcement. In fact, an equilibrium, in which non-announcement is off the equilibrium path and construed as a signal of low quality clearly meets this description. However, the comparison between non-announcement and seed money is less clear-cut and similar to Vesterlund (2003), we cannot rule out the existence of equilibria, in which non-announcement is a signal of higher quality than seed money. Such equilibrium requires that non-announcement emerges as the highest quality signal since $q_U^{S,*} > q_U^{M,*}$ holds in every *SPI* equilibrium. Since $\overline{G}_\infty^{N,*}(q_h, q_U^N, \alpha^N) < \overline{G}_\infty^{S,*}(q_h)$ for $q_U^N < q_h$ (see Lemma A-3), it also requires that seed money induces little verification, preventing the high quality charity from generating significant separation from the low quality charity under seed money. Intuitively, the only advantage of seed money over non-announcement in a large economy is in its signaling potential through the lead donor's gift. Thus, the lack of significant verification on the lead donor's part would remove this advantage of seed money over non-announcement, opening the possibility for non-announcement to emerge as a higher quality signal.[22]

5.3 Warm-glow giving

So far, we have assumed that donors' giving is driven purely by altruistic motives. However, as contended by Andreoni (1988, 1990) many settings involve donors that exhibit "impure altruism." Thus, in this section, we incorporate warm-glow motives as well and show that our main findings extend as long as the altruistic motives for giving are sufficiently strong to engender finite drop-out levels for all wealth "types" in the economy. To illustrate this point, consider the following generalization of donors' preferences:

$$\tilde{u}_i(g_i, G, q) = h(w_i - g_i) + q\tilde{v}(G, g_i) \tag{18}$$

where $\tilde{v}_G(\cdot) > 0, \tilde{v}_{GG}(\cdot) < 0, \tilde{v}_g(\cdot) > 0$, and $\tilde{v}_{gg}(\cdot) < 0$. Thus, donor i cares not only about the total public good provision, but also about being the one providing it. We further assume that $q\tilde{v}_G(0,0) > h'(w_1)$, ensuring positive equilibrium provision; $\tilde{v}_{Gg}(\cdot) + \tilde{v}_{gg}(\cdot) < 0$, ensuring the uniqueness of the follower donors' best response function; and $\tilde{v}_{GG}(\cdot) + \tilde{v}_{gG}(\cdot) < 0$,

[22] While we cannot rule out this possibility, constructing such an equilibrium is challenging. Similar to seed money, it is easy to see that non-announcement cannot be perfectly informative in equilibrium, implying that the two schemes are equally attractive for both quality types, i.e. $\overline{G}_\infty^{N}(q_j, q_U^{N,*}, \alpha^{N,*}) = \overline{G}_{E,\infty}^{S}(q_j, q_U^{S,*}, \alpha^{S,*})$ for $j = \{l, h\}$. This, in turn, requires that $\alpha^{N,*}[\overline{G}_\infty^{N,*}(q_h, q_U^{N,*}, \alpha^{N,*}) - \overline{G}_\infty^{N,*}(q_l, q_U^{N,*}, \alpha^{N,*})] = \alpha^{S,*}[\overline{G}_\infty^{S,*}(q_h) - \overline{G}_\infty^{S,*}(q_l)]$. As revealed by Lemma A-3, the gap in informed contributions between the high and the low quality charity is lower under non-announcement as it precludes information transmission to downstream donors. As a result, non-announcement should result in significantly more information acquisition than seed money. Investigating this possibility numerically using a CES utility function for donors' preferences reveals that this requires very low expected quality under seed money, making matching more attractive than seed money for the low quality charity. This, in turn, causes the *SPI* equilibrium to fail.

ensuring downward sloping reaction functions, which in turn guarantees the uniqueness of the equilibrium contributions. Consistent with our main model, we continue to assume that $\underset{G\to\infty}{lim}\, q\tilde{v}_G(G,0) = 0$, which implies that altruistic motives for giving eventually vanish as provision grows[23]. Then, under common belief about q, donor i's optimal contribution amount solves:

$$h'(w_i - g_i^Z) = q\tilde{v}_G(G^Z, g_i^Z)\,(1 + m\mathbb{1}_M) + q\tilde{v}_g(G^Z, g_i^Z). \tag{19}$$

The right-hand side of eq. (19) is the sum of the marginal benefit of contributing to the public good due to altruism and warm-glow, respectively. Interestingly, only the former depends directly on the match ratio since the matching contributions are given by the lead donor and thus do not induce any additional warm-glow for the follower donors. Therefore, from the above equation, it is evident that the difference between the followers' response to matching and seed money, which is at the core of our analysis in Section 4, is driven entirely by altruistic considerations. Consequently, in order for matching to outperform seed money in a large economy, it is necessary for altruism to outweigh warm-glow considerations by donors, resulting in a finite contribution amount in the limit economy. The following Proposition provides sufficient conditions for matching to strictly outperform seed money in the presence of warm-glow.

Proposition 9 *Given publicly observable quality q, in the limit economy (i.e., $n \to \infty$) total contributions, $\lim_{n\to\infty} \tilde{G}^{Z,*}(q) = \tilde{G}^{Z,*}_\infty(q)$, satisfy:*

- *a) $\tilde{G}^{S,*}_\infty(q) \leq \tilde{G}^{M,*}_\infty(q) < \infty$ if $\lim_{G\to\infty} q\tilde{v}_g(G,0) < h'(w_1)$.[24] Moreover, the first inequality is strict if the elasticity $\epsilon_{\tilde{v}}(\tilde{G}^{S,0}_1(q), 0) < 1$.[25]*
- *b) $\tilde{G}^{S,*}_\infty(q) = \tilde{G}^{M,*}_\infty(q) = \infty$ if $\lim_{G\to\infty} q\tilde{v}_g(G,0) \geq h'(w_1)$.*

Part a) of Proposition 9 provides a sufficient condition for a finite drop-out level under both schemes. It requires that the warm-glow contribution incentives are weaker than the benefit of private consumption at high levels of public good provision. To see the necessity of this condition, recall that the drop-out level $\tilde{G}^{Z,0}_i(q, m\mathbb{1}_M)$ solves

$$q\tilde{v}_G(\tilde{G}^{Z,0}_i, 0)\,(1 + m\mathbb{1}_M) + q\tilde{v}_g(\tilde{G}^{Z,0}_i, 0) - h'(w_i) = 0. \tag{20}$$

Since the altruistic motives for giving vanish as the public good provision grows, i.e. $\underset{G\to\infty}{lim}\, q\tilde{v}_G(G,0) = 0$, in order for contributions to converge to a finite level, it must be the case

[23] Recall from eq. (3), that this ensures finite drop-out level in absence of warm-glow.

[24] Yildirim (2014) shows that this condition is equivalent to downward sloping reaction functions in the limit economy under simultaneous contributions.

[25] Recall that the elasticity of the marginal value $\tilde{v}_G(G,g)$ is $\epsilon_{\tilde{v}}(G,g) = -\frac{(\tilde{v}_{GG}(G,g) + \tilde{v}_{gG}(G,g))G}{\tilde{v}_G(G,g)}$.

that the warm-glow motives for giving are also weak to dissuade giving at large levels of public good provision. Then, analogous to Section 3, finite provision in the limit economy implies that matching outperforms seed money. The comparison between the two schemes is strict as long as the lead donor finds it optimal to provide a positive match in the limit economy, which is guaranteed by the last condition in part a) of Proposition 9. This strict preference for matching by the charity, in turn, sets the stage for the charity to use seed money as a costly signal of high quality. In fact, it is straightforward to verify that our analysis in Section 4 generalizes to preferences that satisfy the conditions outlined in part a).

Part b) of Proposition 9 reveals that in the presence of strong warm-glow motivations, public good provision grows infinitely large as the donor population increases. This occurs because some wealthy donor types, driven purely by warm-glow considerations, give strictly positive individual contributions in the limit economy. As a result, total contributions increase without bound as the donor population grows. This has important implications for the charity's signaling incentives. In particular, with warm-glow as the only driving force, the form of the leadership gift does not have any effect on the followers' giving motives. Thus, matching and seed money become equivalent in a large economy and end up raising the same amount of funds. This makes signaling by the charity obsolete and thus seed money is as likely to be a signal of high quality as matching. Overall, Proposition 9 reveals that altruistic motives for giving play an important role in incentivizing quality signaling via the fundraising scheme in a large economy.

5.4 Alternative information source

In our base model, we assume that the scheme and the lead donor's donation amount are the only possible sources of information for follower donors. This stark case aims to isolate the informational impact of the scheme from other possible sources of information. In this section, we briefly consider the impact of alternative information sources. To capture this possibility in a simple framework, suppose that there is an alternative information channel, which is successful in reaching downstream donors with probability γ. Suppose also that the realization of γ occurs after the information acquisition decision by the lead donor.[26]

Recall from Section 4 that the *SPI* equilibrium requires partial information acquisition, which in turn implies that the value of information under seed money should equal its cost, i.e. $V_I^S(\pi^{S,*}) = k$. However, the alternative information source available to donors should intuitively reduce the value of information for the lead donor, as it reduces the lead donors'

[26]Alternatively, if γ is realized prior to the lead donor's information acquisition and contribution decisions, the two subgames that start after the realization of γ would correspond to either the complete information game described in Section 3 or the incomplete information game described in Section 4. Thus, the probability of seed money fundraising will trivially decrease as γ increases since, as shown in Section 3, the charity has strict preference for matching when donors are exogenously informed about the charity's quality.

need to signal the charity's quality to the follower donors. This is particularly salient in the limit economy, in which the lead donor's sole purpose for information acquisition under seed money is to transmit this information to the follower donors. To see this, recall from Proposition 1 that under seed money, any contribution amount by the lead donor is fully crowded-out, implying that the total money raised, $G_1^{S,0}(q_{\mathcal{F}})$, is only a function of the follower donors' belief about the charity's quality. Thus, given the lead donor's fully separating contribution strategy, $\overline{g}_{L,\infty}^{S,*}(q_{\mathcal{L}})$, her informed and uninformed equilibrium utilities in the limit economy are given by

$$\overline{u}_{L,\infty}^{S}(q_j,\gamma) = h(w_1 - \overline{g}_{L,\infty}^{S,*}(q_j)) + q_j v(G_1^{S,0}(q_j)), \tag{21}$$

$$\begin{aligned} \overline{u}_{L,\infty}^{S}(q_U^S,\gamma) &= h(w_1 - \overline{g}_{L,\infty}^{S,*}(q_U^S)) + (1-\gamma) q_U^S v(G_1^{S,0}(q_U^S)) + \\ &+ \gamma(\pi_h^S q_h v(G_1^{S,0}(q_h)) + \pi_l^S q_l v(G_1^{S,0}(q_l))). \end{aligned} \tag{22}$$

The above utilities capture the fact that the lead donor's gift in the large economy affects total contributions only through its impact on the follower donors' beliefs. Moreover, the uninformed lead donor's impact on total contributions is reduced by the presence of an exogenous information channel. Taking into account the binding incentive constraints for the low and the uninformed types of lead donor given by eq. (11), we arrive at the following observation.

Proposition 10 *The value of information in the limit economy under seed money is given by* $V_{I,\infty}^S(\pi^S,\gamma) = (1-\gamma)\left(q_h - q_U^S\right)\left(v(G_1^{S,0}(q_h)) - v(G_1^{S,0}(q_U^S))\right)\pi_h^S$, *which is strictly decreasing in* γ.

Proposition 10 is a direct consequence of the lead donor's reduced value of signaling as the follower donors become more informed. This, in turn, implies that *SPI* equilibria, which require positive information value by the lead donor, become harder to sustain with more informed donor population. Moreover, the increased possibility of informed giving as a result of the alternative information channel will tilt the charity's preference in favor of the matching scheme. Consequently, consistent with our intuition, the matching scheme should become more prevalent as the reliability of the alternative information channel increases.

6 Concluding remarks

Our analysis provides a theoretical foundation for understanding the recent empirical findings in favor of seed money fundraising. It suggests that seed money can be used as a signaling tool for high quality charities to differentiate themselves from lower quality charities. This conclusion is rather robust since seed money emerges as a signal of higher quality in every equilibrium, in which it is utilized with positive probability and features some information acquisition by the lead donor. We show that this finding continues to hold in richer environments that include arbitrary finite number of types, the possibility of simultaneous fundraising, and the presence of warm-glow motivations for giving.

Apart from providing a theoretical rationale for the emerging field experimental data, our theoretical findings provide promising avenue for future empirical work that can test the use of leadership giving in different information environments. On the experimental side, our findings suggest that the optimal fundraising scheme and the resulting donations vary significantly with the information available to donors. By varying the information available to donors in a lab setting, one can directly test this prediction and obtain further insight into the use of seed money fundraising. On the empirical side, our model further suggests that newer charities may be more eager to seek seed money financing compared to established charities since the former are more likely to have reputation building concerns. Moreover, it predicts that donors would respond differently to an announcement of a matching gift if a charity is less established compared to a more established one. These predictions can be investigated using existing data or a field experiment.

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Appendix

Lemma A-1 *Let $g_L^S < G_1^{S,0}(q)$. Then, total donations $G^{Z,L}(q, d_L^Z) = G_F^Z(q, d_L^Z)(1 + m\mathbb{1}_M) + (1 - m\mathbb{1}_M)g_L^S$ are*

a) *strictly increasing in d_L^Z for all Z;*

b) *strictly increasing in n with $\lim_{n\to\infty} G^{Z,L}(q, d_L^Z) = G_1^{Z,0}(q, m\mathbb{1}_M)$, where $G_1^{M,0}(q, 0) = G_1^{S,0}(q)$ and $\frac{dG_1^{M,0}(q,m)}{dm} > 0$.*[27]

Proof of Lemma A-1 We prove each part in sequence.

a) The proof follows a contradiction argument.[28] In particular, let $d_{L,1}^Z < d_{L,2}^Z$, where $g_{L,2}^S < G_1^{S,0}(q)$ (by assumption), and suppose that $G^{Z,L}(q, d_{L,2}^Z) \leq G^{Z,L}(q, d_{L,1}^Z)$. Letting e_1 and e_2 denote the lowest contributing wealth type under $d_{L,1}^Z$ and $d_{L,2}^Z$ respectively, note that by definition $G^{Z,L}(q, d_{L,1}^Z) < G_{e_1}^{Z,0}(q, \mathbb{1}_M m_1)$. Implicit differentiation of eq. (3) w.r.t. m results in
$$\frac{dG_i^{M,0}(q,m)}{dm} = -\frac{v'(G_i^{M,0})}{v''(G_i^{M,0})(1+m)} > 0 \qquad \text{(A-1)}$$
since $v'(\cdot) > 0$ and $v''(\cdot) < 0$. Therefore, $G^{Z,L}(q, d_{L,2}^Z) \leq G^{Z,L}(q, d_{L,1}^Z) < G_{e_1}^{Z,0}(q, \mathbb{1}_M m_1) \leq G_{e_1}^{Z,0}(q, \mathbb{1}_M m_2)$. It follows that all contributors under $d_{L,1}^Z$ should also be contributors under $d_{L,2}^Z$, which in turn implies that $e_2 \geq e_1$. Then, by Lemma 1, the equilibrium total giving by the follower donors is given by:
$$G_F^Z(q, d_{L,y}^Z) = \sum_{j=1}^{e_y} t_j \left[w_j - \phi\left(qv'(G^{Z,L}(q, d_{L,y}^Z))(1 + m_y\mathbb{1}_M) \right) \right] \; for \; y = \{1, 2\} \qquad \text{(A-2)}$$
where $m_2 > m_1$. Since $\phi(\cdot)$ and $v'(\cdot)$ are both strictly decreasing in their arguments, $e_2 \geq e_1$, $G^{Z,L}(q, d_{L,2}^Z) \leq G^{Z,L}(q, d_{L,1}^Z)$, and $m_2 > m_1$ for $Z = M$, eq. (A-2) requires that $G_F^Z(q, d_{L,2}^Z) > G_F^Z(q, d_{L,1}^Z)$. This, however, leads to a contradiction since $d_{L,2}^Z > d_{L,1}^Z$ and $G_F^Z(q, d_{L,2}^Z) > G_F^Z(q, d_{L,1}^Z)$ imply that $G^{Z,L}(q, d_{L,2}^Z) > G^{Z,L}(q, d_{L,1}^Z)$ for all Z. This establishes that $G^{Z,L}(q, d_L^Z)$ is strictly increasing in d_L^Z for all Z.

b) Proving that $G^{Z,L}(q, d_L^Z)$ is increasing in n is analogous to the proof of part a). Letting $n_2 > n_1 \geq 1$, and e_2 and e_1 denote the corresponding lowest contributing wealth types, suppose that $G^{Z,L}(q, d_L^Z|n_2) \leq G^{Z,L}(q, d_L^Z|n_1) < G_{e_1}^{Z,0}(q, m\mathbb{1}_M)$. The last inequality, in

[27] Recall that n denotes the number of replications of the original economy $\mathcal{D}$.
[28] For $Z = S$, the proof is analogous to the proof of Proposition 3 in Yildirim (2014).

turn, implies that $e_2 \geq e_1$ and the equilibrium total giving by the follower donors is given by:

$$G_F^Z(q, d_L^Z|n_y) = n_y \sum_{j=1}^{e_y} t_j \left[w_j - \phi\left(qv'(G^{Z,L}(q, d_L^Z|n_y))(1 + m\mathbb{1}_M)\right)\right] \ for\ y = \{1, 2\} \quad \text{(A-3)}$$

Then, $n_2 > n_1$, $e_2 \geq e_1$, and $G^{Z,L}(q, d_L^Z|n_2) \leq G^{Z,L}(q, d_L^Z|n_1)$ immediately imply that $G_F^Z(q, d_L^Z|n_2) > G_F^Z(q, d_L^Z|n_1)$. This, in turn results in a contradiction since $G^{Z,L}(q, d_L^Z|n)$ is strictly increasing in $G_F^Z(q, d_L^Z|n)$, implying that $G^{Z,L}(q, d_L^Z|n_2) > G^{Z,L}(q, d_L^Z|n_1)$. Therefore, $G^{Z,L}(q, d_L^Z)$ is strictly increasing in n.

To prove that $\lim_{n\to\infty} G^{Z,L}(q, d_L^Z) = G_1^{Z,0}(q, \mathbb{1}_M m)$, first note that $\lim_{n\to\infty} G^{Z,L}(q, d_L^Z) > G_1^{Z,0}(q, \mathbb{1}_M m)$ implies that $\lim_{n\to\infty} C^Z = \emptyset$ and $G^{Z,L}(q, d_L^Z) = (1 - \mathbb{1}_M)g_L^S < G_1^{Z,0}(q, \mathbb{1}_M m)$, a contradiction. Alternatively, $\lim_{n\to\infty} G^{Z,L}(q, d_L^Z) < G_1^{Z,0}(q, \mathbb{1}_M m)$ implies that a follower of wealth w_1 contributes a strictly positive amount, i.e. $\lim_{n\to\infty} g_1^Z(q, d_L^Z) > 0$, which in turn leads to $\lim_{n\to\infty} G^{Z,L}(q, d_L^Z) \geq \lim_{n\to\infty} nt_1 g_1^Z(q, d_L^Z) = \infty$, a contradiction. This establishes that $\lim_{n\to\infty} G^{Z,L}(q, d_L^Z) = G_1^{Z,0}(q, \mathbb{1}_M m)$. Finally, $G_1^{M,0}(q, 0) = G_1^{S,0}(q)$ follows immediately from eq. (3) and by eq. (A-1) $\frac{dG_1^{M,0}(q,m)}{dm} > 0$.

■

Proof of Proposition 1

Lemma A-1 proves the equilibrium properties of $G^{Z,L}(q, d_L^Z)$. Thus, we focus on establishing the equilibrium impact of d_L^Z on $G_F^Z(q, d_L^Z)$.

To prove part a), first note that by Lemma A-1, $g_{L,2}^S > g_{L,1}^S$ implies that $G^{S,L}(q, g_{L,1}^S) < G^{S,L}(q, g_{L,2}^S) < G_{e_2}^{S,0}(q)$, where e_i for $i = \{1, 2\}$ denotes the lowest contributing wealth type under $g_{L,i}^S$. Thus, $e_2 \leq e_1$. Then, by eq. (A-2), $G^{S,L}(q, g_{L,2}^S) > G^{S,L}(q, g_{L,1}^S)$ and $e_2 \leq e_1$ immediately imply that $G_F^Z(q, g_{L,2}^Z) < G_F^Z(q, g_{L,1}^Z)$ since $\phi(\cdot)$ and $v'(\cdot)$ are both strictly decreasing in their arguments.

To prove part b), let $g_i^M(q, m)$ denote the solution to eq. (2) for a fixed m, which corresponds to total giving $G^{M,L}(q, m)$. Then, by implicit differentiation of eq. (2),

$$\frac{dg_i^M(q,m)}{dm} = -\phi'(qv'(G^{M,L}(q, m))(1 + m))q\left(v'(G^{M,L}) + v''(G^{M,L})(1 + m)\frac{dG^{M,L}(q,m)}{dm}\right) \quad \text{(A-4)}$$

Eq. (A-4) reveals that $\frac{dg_i^M(q,m)}{dm}$ is independent of w_i and thus identical across all donors with $g_i^M(q, m) \geq 0$. Let N_m denote the number of follower donors with $g_i^M(q, m) \geq 0$ given m. Then, by definition, $\frac{dG_F^M(q,m)}{dm} = N_m \frac{dg_i^M(q,m)}{dm}$. Moreover, recall that $G^{M,L}(q, m) = (1 + m)G_F^M(q, m)$. Therefore, $\frac{dG^{M,L}(q,m)}{dm} = G_F^M(q, m) + (1 + m)\frac{dG_F^M(q,m)}{dm} = G_M^F(q, m) + (1 + m)N_m \frac{dg_i^M(q,m)}{dm}$. Substituting for $\frac{dG^{M,L}(q,m)}{dm}$ in eq. (A-4), and solving for $\frac{dg_i^M(q,m)}{dm}$ obtains:

$$\frac{dg_i^M(q,m)}{dm} = \frac{-\phi'\left(qv'(G^{M,L}(q, m))(1 + m)\right)q\left(v'(G^{M,L}) + v''(G^{M,L})G^{M,L}\right)}{1 + \phi'\left(qv'(G^{M,L}(q, m)(1 + m)\right)qv''(G^{M,L})(1 + m)^2 N_m} \quad \text{(A-5)}$$

Since $\phi'(\cdot) < 0$ and $v''(\cdot) < 0$, by eq. (A-5), $\frac{dg_i^M(q,m)}{dm} > 0$ (and thus $\frac{dG_F^M(q,m)}{dm} > 0$) if and only if $\frac{-v''(G^{M,L})G^{M,L}}{v'(G^{M,L})} = \epsilon_v(G^{M,L}) < 1$. ■

Proof of Proposition 2

Let $G_\infty^Z(q, d_L^Z) = \underset{n\to\infty}{lim} G^{Z,L}(q, d_L^Z)$ and $G_\infty^{Z,*}(q) = \underset{n\to\infty}{lim} G^{Z,L}(q, d_L^{Z,*})$ where $d_L^{Z,*}(q)$ denotes the equilibrium value of d_L^Z. Then, by Proposition 1, $G_\infty^Z(q, d_L^Z) = G_1^{Z,0}(q, m\mathbb{1}_M)$. For $Z = S$, $G_\infty^S(q, g_L^Z) = G_1^{S,0}(q) = G_\infty^{S,*}(q)$ since $G_1^{S,0}(q)$ does not depend on g_L^S. By Lemma A-1, $G_\infty^M(q,m) = G_1^{M,0}(q,m) \geq G_1^{S,0}(q)$ with strict inequality for $m > 0$. To complete the proof, we show that $\lim_{n\to\infty} m^*(q) > 0$ if $\frac{-v''(G_1^{S,0}(q))G_1^{S,0}(q)}{v'(G_1^{S,0}(q))} = \epsilon_v(G_1^{S,0}(q)) < 1$. Note that $\frac{du_L(q,0)}{dm} > 0$ guarantees $m^*(q) > 0$. In the limit economy, by eq. (7), $\lim_{n\to\infty} \frac{du_L(q,0)}{dm} = h'(w_1)\lim_{n\to\infty} \frac{dG_F^M(q,0)}{dm}$ since $G_1^{S,0}(q)$ solves eq. (3). Recall that by definition $G_F^M(q,m) = \frac{G^{M,L}(q,m)}{1+m}$. Since $\lim_{n\to\infty} G^{M,L}(q,m) = G_1^{M,0}(q,m)$, $\lim_{n\to\infty} G_F^M(q,m) = \frac{G_1^{M,0}(q,0)}{1+m}$. Therefore, $\lim_{n\to\infty} \frac{dG_F^M(q,m)}{dm} = \frac{1}{1+m}\frac{dG_1^{M,0}(q,m)}{dm} - \frac{G_1^{M,0}(q,m)}{(1+m)^2}$. Substituting for $\frac{dG_1^{M,0}(q,m)}{dm}$ from eq. (A-1) and evaluating at $m = 0$ obtains

$$\lim_{n\to\infty} \frac{dG_F^M(q,0)}{dm} = G_1^{M,0}(q,0)\left(\frac{v'(G_1^{M,0}(q,0))}{-v''(G_1^{M,0}(q,0))G_1^{M,0}(q,0)} - 1\right) \tag{A-6}$$

Taking into account that $G_1^{M,0}(q,0) = G_1^{S,0}(q)$ (see Lemma A-1), eq. (A-6) implies that $\lim_{n\to\infty} \frac{du_L(q,0)}{dm} = h'(w_1)\lim_{n\to\infty} \frac{G_F^M(q,0)}{dm} > 0$ if and only if $\epsilon_v(G_1^{S,0}(q)) < 1$. This completes the proof. ■

Proof of Lemma 2

Let $\overline{G}_\infty^{Z,*}(q^Z) = \underset{n\to\infty}{lim}\overline{G}^{Z,*}(q^Z)$. To prove the statement in Lemma 2, it suffices to show that there exists $\overline{n} < \infty$, such that $\overline{G}^{M,*}(q^M) > \overline{G}^{S,*}(q^S)$ for $q^M \geq q^S$ and $n > \overline{n}$. Note that by Proposition 1,

$$\overline{G}_\infty^{S,*}(q^S) = \lim_{n\to\infty} G^{S,L}(q^S, \overline{g}_L^{S,*}) = G_1^{S,0}(q^S). \tag{A-7}$$

Moreover, implicitly differentiating eq. (3) w.r.t. q^S for $Z = S$ results in $\frac{dG_1^{S,0}(q^S)}{dq^S} = -\frac{v'(G_1^{S,0}(q^S))}{qv''(G_1^{S,0}(q^S))} > 0$, implying that $G_1^{S,0}(q^M) > G_1^{S,0}(q^S)$.

Turning to $Z = M$, note that the optimization problem given by eq. (11) implies that $\overline{m}^*(q^M) \geq m^*(q^M) > 0$ (see also Claim 1 in the proof of Lemma A-2). Then, by Proposition 1,

$$\overline{G}_\infty^{M,*}(q^M) = G_1^{M,0}(q^M, \overline{m}^*(q^M)) \geq G_1^{M,0}(q^M, m^*(q^M)) = G_\infty^{M,*}(q^M) \tag{A-8}$$

since by eq. (A-1), $G_1^{M,0}(q^M, m)$ is strictly increasing in m. By Lemma A-1,$G_1^{M,0}(q^M, m^*(q^M)) > G_1^{S,0}(q^M)$ since $m^*(q^M) > 0$. Thus, (A-7) and (A-8) imply that $\overline{G}_\infty^{M,*}(q^M) > \overline{G}_\infty^{S,*}(q^M) > \overline{G}_\infty^{S,*}(q^S)$.

To complete the proof, it suffices that $\overline{G}^{Z,*}(q^Z)$ is continuous in n. This follows from the fact that $G_F^Z(q_{\mathcal{F}}^Z, d_L^Z)$ is continuous in n (by eq. (A-3)), implying the continuity of $\overline{u}_L(q^Z, q_{\mathcal{F}}^Z, d_L^Z)$. Therefore, $\overline{G}_\infty^{M,*}(q^M) > \overline{G}_\infty^{S,*}(q^S)$ implies that there exists $\overline{n} < \infty$ such that $\overline{G}^{M,*}(q^M) > \overline{G}^{S,*}(q^S)$ for all $n > \overline{n}$. ■

Lemma A-2 *Let $\mathcal{Q}_L^Z = \{q_1, q_2, ..., q_y\}$ with $q_j > q_{j-1}$ for all $j \in \mathbb{Z} : j \in [2, y]$ denote the set of quality types of the lead donor. Moreover, the equilibrium donation $\overline{d}_L^{Z,*}(q_j)$ by each type $q_j \in \mathcal{Q}_L^Z$ satisfies*

$$\begin{aligned} &\overline{d}_L^{Z,*}(q_j) = \underset{d_L^Z}{argmax}\, \overline{u}_L(q_j, q_j, d_L^Z) \\ s.t. \quad &\overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j)) \geq \overline{u}_L(q_j, \tilde{q}, \overline{d}_L^{Z,*}(\tilde{q}))\ \text{for all}\ q_j, \tilde{q} \in \mathcal{Q}_L^Z \end{aligned} \tag{A-9}$$

Then, $\overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j)) > \overline{u}_L(q_j, \tilde{q}, \overline{d}_L^{Z,*}(\tilde{q}))$ for all $q_j \in \mathcal{Q}_L^Z$ and all $\tilde{q} \in \mathcal{Q}_L^Z \setminus \{q_j, q_{j+1}\}$.

Proof Note that $\overline{G}^{Z,*}(q_j) = G^{Z,L}(q_j, \overline{d}_L^{Z,*}(q_j))$ and let $\overline{g}_L^{Z,*}(q_j)$ denote the equilibrium donation by L, where by definition $\overline{g}_L^{S,*}(q_j) = \overline{d}_L^{S,*}(q_j)$ and $\overline{g}_L^{M,*}(q_j) = \overline{d}_L^{M,*}(q_j)\overline{G}_F^{M,*}(q_j, \overline{d}_L^{M,*}(q_j))$. The proof proceeds by establishing three claims.

- **Claim 1**: The equilibrium total donation $\overline{G}^{Z,*}(q_j)$ and the lead donor's gift $\overline{g}_L^{Z,*}(q_j)$ are strictly increasing in j.

  We first establish that $\overline{G}^{Z,*}(q_j)$ is strictly increasing in j by means of a contradiction. Contrary to Claim 1, suppose that there exists $j \in [2, y]$ such that $\overline{G}^{Z,*}(q_{j-1}) \geq \overline{G}^{Z,*}(q_j)$. The incentive constraint for q_{j-1} given by eq. (A-9) requires

$$\begin{aligned} &\overline{u}_L(q_{j-1}, q_{j-1}, \overline{d}_L^{Z,*}(q_{j-1})) \geq \overline{u}_L(q_{j-1}, q_j, \overline{d}_L^{Z,*}(q_j)) \Longrightarrow \\ &q_{j-1}\left[v(\overline{G}^{Z,*}(q_{j-1})) - v(\overline{G}^{Z,*}(q_j))\right] \geq h(w_L - \overline{g}_L^{Z,*}(q_j)) - h(w_L - \overline{g}_L^{Z,*}(q_{j-1})) \end{aligned} \tag{A-10}$$

  Consider first the possibility of $\overline{G}^{Z,*}(q_{j-1}) = \overline{G}^{Z,*}(q_j)$. Then, the inequality given by (A-10) becomes $h(w_L - \overline{g}_L^{Z,*}(q_j)) - h(w_L - \overline{g}_L^{Z,*}(q_{j-1})) \leq 0$, which in turn implies that $\overline{g}_L^{Z,*}(q_j) > \overline{g}_L^{Z,*}(q_{j-1})$ in a separating equilibrium since $h(\cdot)$ is strictly increasing in its argument.[29] It immediately follows that $\overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j)) < \overline{u}_L(q_j, q_{j-1}, \overline{d}_L^{Z,*}(q_{j-1}))$,

[29]For $Z = S$, it is immediately obvious that $\overline{g}_L^{S,*}(q_j) \neq \overline{g}_L^{S,*}(q_{j-1})$ in a separating equilibrium. For $Z = M$, note

which violates eq. (A-9) for q_j, and contradicts $\overline{g}_L^{Z,*}(q_j)$ being an optimal solution. Therefore, $\overline{G}^{Z,*}(q_{j-1}) = \overline{G}^{Z,*}(q_j)$ cannot occur in a separating equilibrium.

Next, consider $\overline{G}^{Z,*}(q_{j-1}) > \overline{G}^{Z,*}(q_j)$. Then, $[q_j - q_{j-1}]\left[v(\overline{G}^{Z,*}(q_{j-1})) - v(\overline{G}^{Z,*}(q_j))\right] > 0$ since $v(\cdot)$ is increasing in its argument. Therefore, by (A-10), this implies that

$$q_j\left[v(\overline{G}^{Z,*}(q_{j-1})) - v(\overline{G}^{Z,*}(q_j))\right] > h(w_L - \overline{g}_L^{Z,*}(q_j)) - h(w_L - \overline{g}_L^{Z,*}(q_{j-1})) \Longrightarrow$$
$$\overline{u}_L(q_j, q_{j-1}, \overline{d}_L^{Z,*}(q_{j-1})) > \overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j))$$

The last inequality violates eq. (A-9) for q_j, and again contradicts $\overline{g}_L^{Z,*}(q_j)$ being an optimal solution. Therefore, $\overline{G}^{Z,*}(q_{j-1}) > \overline{G}^{Z,*}(q_j)$ cannot occur in a separating equilibrium. This, in turn, implies that $\overline{G}^{Z,*}(q_j)$ is strictly increasing in q_j.

To establish that $\overline{g}_L^{Z,*}(q_j)$ is strictly increasing in j, note that $\overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j))$ is strictly increasing in $\overline{G}^{Z,*}(q_j)$ and strictly decreasing in $\overline{g}_L^{Z,*}(q_j)$. Therefore, $\overline{g}_L^{Z,*}(q_j) \geq \overline{g}_L^{Z,*}(q_{j+1})$ for some j would result in $\overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j)) < \overline{u}_L(q_j, q_{j+1}, \overline{d}_L^{Z,*}(q_{j+1}))$ since $\overline{G}^{Z,*}(q_{j+1}) > \overline{G}^{Z,*}(q_j)$. This, in turn, violates eq. (A-9). Therefore, $\overline{g}_L^{Z,*}(q_j)$ must be strictly increasing in j. This completes the proof of Claim 1.

- **Claim 2** Let $|z| \geq 1$ and $a = \frac{z}{|z|}$. Then, $\overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j)) \geq \overline{u}_L(q_j, q_{j-z}, \overline{d}_L^{Z,*}(q_{j-z}))$ for any j implies $\overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j)) > \overline{u}_L(q_j, q_{j-z-a}, \overline{d}_L^{Z,*}(q_{j-z-a}))$.

  Note that by definition,

$$\overline{u}_L(q_j, q_{j-z}, \overline{d}_L^{Z,*}(q_{j-z})) = \overline{u}_L(q_{j-z}, q_{j-z}, \overline{d}_L^{Z,*}(q_{j-z})) + (q_j - q_{j-z})v(\overline{G}^{Z,*}(q_{j-z})). \quad \text{(A-11)}$$

  Then, by eq. (A-9),

$$\begin{aligned}
\overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j)) &\geq \overline{u}_L(q_{j-z}, q_{j-z}, \overline{d}_L^{Z,*}(q_{j-z})) + (q_j - q_{j-z})v(\overline{G}^{Z,*}(q_{j-z})) \geq \\
&\geq \overline{u}_L(q_{j-z}, q_{j-z-a}, \overline{d}_L^{Z,*}(q_{j-z-a})) + (q_j - q_{j-z})v(\overline{G}^{Z,*}(q_{j-z})) = \\
&= \overline{u}_L(q_j, q_{j-z-a}, \overline{d}_L^{Z,*}(q_{j-z-a})) + (q_j - q_{j-z})\left[v(\overline{G}^{Z,*}(q_{j-z})) - v(\overline{G}^{Z,*}(q_{j-z-a}))\right]
\end{aligned}$$

  It follows from the above equation that $\overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j)) > \overline{u}_L(q_j, q_{j-z-a}, \overline{d}_L^{Z,*}(q_{j-z-a}))$ since $v'(G) > 0$ and by Claim 1, $\overline{G}^{Z,*}(q_j)$ is strictly increasing in q_j, implying that $(q_j - q_{j-z})\left[v(\overline{G}^{Z,*}(q_{j-z})) - v(\overline{G}^{Z,*}(q_{j-z-a}))\right] > 0$.

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that $\overline{G}^{M,*}(q_j) = \overline{G}_F^{M,*}(q_j, \overline{m}^*(q_j)) + \overline{m}^*(q_j)\overline{G}_F^{M,*}(q_j, \overline{m}^*(q_j))$. Since $G_F^M(q_j, m)$ is strictly increasing in m and q_j, it follows that $\overline{g}_L^{Z,*}(q_j) = \overline{g}_L^{Z,*}(q_{j-1})$ requires that $\overline{m}^*(q_j) < \overline{m}^*(q_{j-1})$ and $\overline{G}_F^{M,*}(q_j, \overline{m}^*(q_j)) > \overline{G}_F^{M,*}(q_{j-1}, \overline{m}^*(q_{j-1}))$. This, however, results in $\overline{G}^{M,*}(q_j) > \overline{G}^{M,*}(q_{j-1})$, contradicting our conjecture of $\overline{G}^{M,*}(q_j) = \overline{G}^{M,*}(q_{j-1})$. Thus, $\overline{g}_L^{M,*}(q_j) \neq \overline{g}_L^{M,*}(q_{j-1})$.

It follows by Claim 2 that all constraints with $|z| > 1$ are non-binding as they are implied by $|z| = 1$. The third, and final, claim shows that the constraint for $z = 1$ is non-binding as well, leaving $z = -1$ as the only possible binding constraint.

- **Claim 3** $\overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j)) > \overline{u}_L(q_j, q_{j-1}, \overline{d}_L^{Z,*}(q_{j-1}))$ for all $j \in [2, y]$.

  Let $D_j(\overline{d}_L^{Z,*}(q_j), \overline{d}_L^{Z,*}(q_{j-1})) = \overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j)) - \overline{u}_L(q_j, q_{j-1}, \overline{d}_L^{Z,*}(q_{j-1}))$. By eq. (A-11),

$$\begin{aligned} D_{j-1}(\overline{d}_L^{Z,*}(q_{j-1}), \overline{d}_L^{Z,*}(q_j)) \quad &+ \quad D_j(\overline{d}_L^{Z,*}(q_j), \overline{d}_L^{Z,*}(q_{j-1})) = \\ &= \quad (q_j - q_{j-1}) \left[v(\overline{G}^{Z,*}(q_j)) - v(\overline{G}^{Z,*}(q_{j-1})) \right] > 0 \end{aligned} \tag{A-12}$$

  where the strict inequality follows from Claim 1. Contrary to Claim 3, suppose that there exists $k \in [2, y]$ such that $D_k(\overline{d}_L^{Z,*}(q_k), \overline{d}_L^{Z,*}(q_{k-1})) = 0$. By eq. (A-12), $D_{k-1}(\overline{d}_L^{Z,*}(q_{k-1}), \overline{d}_L^{Z,*}(q_k)) > 0$. We next show that there exists alternative contribution levels $\tilde{d}_L^Z(q_j)$ for all q_j that satisfy (A-9) and result in $u_L(q_j, q_j, \tilde{d}_L^Z(q_j)) \geq u_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j))$, with a strict inequality for $j = k$. This contradicts the optimality of $\overline{d}_L^{Z,*}(q_j)$. To establish the existence of such $\tilde{d}_L^Z(q_j)$, let $\tilde{d}_L^Z(q_j)$ satisfy the following properties:

  1. for $j < k$, $\tilde{d}_L^Z(q_j) = \overline{d}_L^{Z,*}(q_j)$.
  2. for $j \geq k$, we define recursively $\tilde{d}_L^Z(q_j)$ such that if $D_j(\overline{d}_L^{Z,*}(q_j), \tilde{d}_L^Z(q_{j-1})) \leq 0$, $\tilde{d}_L^Z(q_j)$ solves $D_{j-1}(\tilde{d}_L^Z(q_{j-1}), \tilde{d}_L^Z(q_j)) = 0$; otherwise $\tilde{d}_L^Z(q_j) = \overline{d}_L^{Z,*}(q_j)$.

  For $D_j(\overline{d}_L^{Z,*}(q_j), \tilde{d}_L^Z(q_{j-1})) > 0$, it follows trivially that $u_L(q_j, q_j, \tilde{d}_L^Z(q_j)) = u_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j))$. For $D_j(\overline{d}_L^{Z,*}(q_j), \tilde{d}_L^Z(q_{j-1})) \leq 0$, eq. (A-12) implies that $D_{j-1}(\tilde{d}_L^Z(q_{j-1}), \overline{d}_L^{Z,*}(q_j)) > 0$. Therefore, by the continuity of $\overline{G}^{Z,L}(q_j, d_L^Z)$ and $\overline{g}_L^Z(q_j, d_L^Z)$ in d_L^Z, there exists $\tilde{d}_L^Z(q_j)$ with resulting total contributions $\overline{G}^{Z,L}(q_j, \tilde{d}_L^Z(q_j)) \in \left(\overline{G}^{Z,L}(q_{j-1}, \tilde{d}_L^Z(q_{j-1})), \overline{G}^{Z,*}(q_j)\right)$ satisfying $D_{j-1}(\tilde{d}_L^Z(q_{j-1}), \tilde{d}_L^Z(q_j)) = 0$. Moreover, by eq. (A-12), $D_j(\tilde{d}_L^Z(q_j), \tilde{d}_L^Z(q_{j-1})) > 0 = D_j(\overline{d}_L^{Z,*}(q_j), \tilde{d}_L^Z(q_{j-1}))$, implying that $\overline{u}_L(q_j, q_j, \tilde{d}_L^Z(q_j)) > \overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j))$. This establishes that $u_L(q_j, q_j, \tilde{d}_L^Z(q_j)) \geq u_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j))$, with a strict inequality for $j = k$ since by construction $D_k(\overline{d}_L^{Z,*}(q_k), \tilde{d}_L^Z(q_{k-1})) = 0$.

  It remains to establish that $\tilde{d}_L^Z(q_j)$ satisfies (A-9) for all q_j. By Claim 2, it suffices to show that $D_j(\tilde{d}_L^Z(q_j), \tilde{d}_L^Z(q_{j+z})) \geq 0$ for $z = \{-1, 1\}$. Consider first $z = -1$. Then, if $\tilde{d}_L^Z(q_j) = \overline{d}^{Z,*}(q_j)$, $D_j(\tilde{d}_L^Z(q_j), \tilde{d}_L^Z(q_{j-1})) \geq 0$ follows immediately for $j < k$ since $\tilde{d}_L^Z(q_{j-1}) = \overline{d}_L^{Z,*}(q_{j-1})$ and by assumption $\overline{d}_L^{Z,*}(q_j)$ satisfies eq. (A-9). For $j \geq k$, if $\tilde{d}_L^Z(q_j) = \overline{d}^{Z,*}(q_j)$, then by definition $D_j(\overline{d}_L^{Z,*}(q_j), \tilde{d}_L^Z(q_{j-1})) > 0$. For $j \geq k$ and $\tilde{d}_L^Z(q_j) \neq \overline{d}^{Z,*}(q_j)$, $D_{j-1}(\tilde{d}_L^Z(q_{j-1}), \tilde{d}_L^Z(q_j)) = 0$ and eq. (A-12) implies $D_j(\tilde{d}_L^Z(q_j), \tilde{d}_L^Z(q_{j-1})) > 0$. Turning to $z = 1$, by construction $D_j(\tilde{d}_L^Z(q_j), \tilde{d}_L^Z(q_{j+1})) = 0$ whenever $D_{j+1}(\overline{d}_L^{Z,*}(q_{j+1}), \tilde{d}_L^Z(q_j)) \leq$

0. Otherwise, if $D_{j+1}(\overline{d}_L^{Z,*}(q_{j+1}), \tilde{d}_L^Z(q_j)) > 0$, then $\tilde{d}_L^Z(q_{j+1}) = \overline{d}_L^{Z,*}(q_{j+1})$, which implies that $D_j(\overline{d}_L^{Z,*}(q_j), \tilde{d}_L^Z(q_{j+1})) \geq 0$ since by our assumption $\overline{d}_L^{Z,*}(q_j)$ satisfies eq. (A-9). However, since we have established that $\overline{u}_L(q_j, q_j, \tilde{d}_L^Z(q_j)) \geq \overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j))$, this implies that $D_j(\tilde{d}_L^Z(q_j), \tilde{d}_L^Z(q_{j+1})) \geq D_j(\overline{d}_L^{Z,*}(q_j), \tilde{d}_L^Z(q_{j+1})) \geq 0$.

This establishes that $D_k(\overline{d}_L^{Z,*}(q_k), \overline{d}_L^{Z,*}(q_{k-1})) = 0$ for some $k \in [2, y]$ contradicts the optimality of $\overline{d}_L^{Z,*}(q_k)$. Therefore, for all j, $\overline{d}_L^{Z,*}(q_j)$ must satisfy $\overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j)) > \overline{u}_L(q_j, q_{j-1}, \overline{d}_L^{Z,*}(q_{j-1}))$.

■

Proof of Lemma 3

Recall that $\overline{u}_L^{Z,*}(q_{\mathcal{L}}^Z) = \overline{u}_L(q_{\mathcal{L}}^Z, q_{\mathcal{L}}^Z, \overline{d}_L^{Z,*}(q_{\mathcal{L}}^Z))$, allowing us to re-write eq. (13) as

$$V_I^Z(\pi^Z) = \sum_{j=\{l,h\}} \pi_j^Z \left[\overline{u}_L(q_j, q_j, \overline{d}_L^{Z,*}(q_j)) - \overline{u}_L(q_j, q_U^Z, \overline{d}_L^{Z,*}(q_U^Z)) \right] \tag{A-13}$$

Clearly, $V_I^Z(\pi^Z)$ is continuous in π_h^Z since $\overline{d}_L^{Z,*}(q_U^Z)$ and q_U^Z are continuous functions. Moreover, if $\pi_j^Z = 1$ for some j, then $q_U^Z = q_j$. Thus, $V_I^Z(1,0) = V_I^Z(0,1) = 0$ follows immediately from $\pi_h^Z + \pi_l^Z = 1$. If $\pi_j^Z \in (0,1)$, $V_I^Z(\pi^Z) > 0$ since by Lemma A-2, $\overline{u}_L(q_l, q_l, \overline{d}_L^{Z,*}(q_l)) \geq \overline{u}_L(q_l, q_U^Z, \overline{d}_L^{Z,*}(q_U^Z))$ and $\overline{u}_L(q_h, q_h, \overline{d}_L^{Z,*}(q_h)) > \overline{u}_L(q_h, q_U^Z, \overline{d}_L^{Z,*}(q_U^Z))$. ■

Proof of Proposition 3

Let $\alpha^{Z,*} = 0$ for all Z. By eq. (12), it is immediately obvious that $\overline{G}_E^{Z,*}(q_h, q_U^{Z,*}, 0) = \overline{G}_E^{Z,*}(q_l, q_U^{Z,*}, 0)$ for all Z. Moreover, by definition $\beta^{M,*}(q_j) = 1 - \beta^{S,*}(q_j)$ with $\beta^{Z,*}(q_j) = \underset{\beta^Z}{argmax} \sum_Z \beta^Z \overline{G}_E^{Z,*}(q_j, q_U^{Z,*}, 0)$.

The linearly of the above objective function implies that $\beta^{Z,*}(q_j) \in (0,1)$ if and only if $\overline{G}_E^{M,*}(q_j, q_U^{M,*}, 0) = \overline{G}_E^{S,*}(q_j, q_U^{S,*}, 0)$. Together with $\overline{G}_E^{Z,*}(q_h, q_U^{Z,*}, 0) = \overline{G}_E^{Z,*}(q_l, q_U^{Z,*}, 0)$, this implies that $\overline{G}_E^{M,*}(q_h, q_U^{M,*}, 0) = \overline{G}_E^{M,*}(q_l, q_U^{M,*}, 0) = \overline{G}_E^{S,*}(q_l, q_U^{S,*}, 0) = \overline{G}_E^{S,*}(q_h, q_U^{S,*}, 0)$, completing the proof. ■

Proof of Proposition 4

First, we show that $\alpha^{Z,*} = 1$ for all Z in the equilibrium path implies $\beta^{S,*}(q_j) = 0$ for all $j \in \{l, h\}$. By means of a contradiction argument, suppose that $\beta^{S,*}(q_j) > 0$ for some j. Then, $\alpha^{S,*} = 1$ implies that $V_I^S(\pi^{S,*}) \geq k > 0$. Then by Lemma 3, $\pi_j^{S,*} \in (0,1)$ for all j, which by eq. (9) requires $\beta^{S,*}(q_j) > 0$ for all j. Moreover, by eq. (12) the expected equilibrium contributions are $\overline{G}_E^{S,*}(q_j, q_U^{S,*}, 1) = \overline{G}^{S,*}(q_j)$. However, since $\overline{G}^{S,*}(q_l) < \overline{G}^{M,*}(q_l) \leq \overline{G}_E^{M,*}(q_l, q_U^{M,*}, \alpha^{M,*})$, in such equilibrium $\beta^{S,*}(q_l) = 0$, which in turn implies that $\pi_h^{S,*} = 1$ and $V_I^S(\pi^{S,*}) = 0 < k$, contradicting $\alpha^{S,*} = 1$. Therefore, $\alpha^{S,*} = 1$ requires $\beta^{S,*}(q_j) = 0$ for all j.

To establish the existence of an equilibrium with $\alpha^{M,*} = 1$, note that by eq. (9), $\beta^{M,*}(q_j) = 1 - \beta^{S,*}(q_j) = 1$ for all j implies that $\pi^{M,*} = \pi$. Therefore, $\alpha^{M,*} = 1$ requires $V_I^M(\pi) \geq k$. No deviation incentives to $\beta^S(q_j) > 0$ for some j is guaranteed by an off-equilibrium belief $\alpha^{S,*} = 1$ since $\overline{G}^{S,*}(q_j) < \overline{G}^{M,*}(q_j)$ for all j. ■

Proof of Lemma 4

We first show the existence of a *SPI* equilibrium for $V_I^S(\pi) \geq k$ and $\overline{G}^{S,*}(E[q]) > \overline{G}^{M,*}(q_l)$ by constructing such an equilibrium. By Lemma 3, $V_I^S(\pi^S)$ is continuous in π_h^S and reaches a minimum at $\pi_h^S = 1$ with $V_I^S(0,1) = 0$. This implies that there exists $\breve{\pi}^S$ with $\breve{\pi}_h^S > \pi_h$ such that $V_I^S(\breve{\pi}^S) = k$. Consider an equilibrium with $\pi^{S,*} = \breve{\pi}^S$ and $\beta^{S,*}(q_h) = 1$. Then, by eq. (9), $\beta^{S,*}(q_l) = \frac{\pi_h}{\pi_l}\left(\frac{1}{\breve{\pi}_h} - 1\right) \leq 1$ and $\pi_l^{M,*} = 1$. It follows that $q_U^{M,*} = q_l$, and $V_I^M(\pi^{M,*}) = 0$ (by Lemma 3), implying that $\alpha^{M,*} = 0$. Then, by eq. (12), $\overline{G}_E^{M,*}(q_j, q_U^{M,*}, 0) = \overline{G}^{M,*}(q_l)$ for all j. Since $V_I^S(\pi^{S,*}) = k$, the lead donor is indifferent in her information acquisition strategy α^S. To prevent deviation from $\beta^{S,*}(q_l)$, it suffices that $\overline{G}_E^{S,*}(q_l, q_U^{S,*}, \alpha^{S,*}) = \overline{G}^{M,*}(q_l)$. Substituting for $\overline{G}_E^{S,*}(q_l, q_U^{S,*}, \alpha^{S,*})$ in eq. (12) and solving for $\alpha^{S,*}$ results in $\alpha^{S,*} = \frac{\overline{G}^{S,*}(q_U^{S,*}) - \overline{G}^{M,*}(q_l)}{\overline{G}^{S,*}(q_U^{S,*}) - \overline{G}^{S,*}(q_l)} \in (0,1)$ since $q_U^{S,*} > E[q]$ as a result of $\pi_h^{S,*} > \pi_h$, which implies that $\overline{G}^{S,*}(q_U^{S,*}) > \overline{G}^{S,*}(E[q]) > \overline{G}^{M,*}(q_l) > \overline{G}^{S,*}(q_l)$. Moreover, there is no incentive to deviate from $\beta^{S,*}(q_h)$ since $\overline{G}_E^{S,*}(q_h, q_U^{S,*}, \alpha^{S,*}) \geq \overline{G}_E^{S,*}(q_l, q_U^{S,*}, \alpha^{S,*}) = \overline{G}^{M,*}(q_l)$.

To establish property 1), note that if $\beta^{S,*}(q_j) = 0$ for some j then by eq. (9), $\pi_j^{S,*} = 0$ and thus $V_I^S(\pi^{S,*}) = 0$ with $\alpha^{S,*} = 0$ and $\alpha^{M,*} > 0$ (by Definition 1). Then, $q_U^{S,*} = q_{-j}$ where $q_{-j} = \{l, h\} \setminus \{j\}$ and by eq. (12), $\overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*}) = \overline{G}^{S,*}(q_{-j})$ for all j. If $q_j = q_h$ and $q_{-j} = q_l$, then there is strict deviation incentives to $\beta^M(q_l) = 1$ since $\overline{G}^{S,*}(q_l) < \overline{G}^{M,*}(q_l)$. If $q_j = q_l$ and $q_{-j} = q_h$, $\beta^S(q_h) > 0$ implies that $\overline{G}^{S,*}(q_h) \geq \overline{G}_E^{M,*}(q_h, q_U^{M,*}, \alpha^{M,*}) > \overline{G}_E^{M,*}(q_l, q_U^{M,*}, \alpha^{M,*})$ due to the fact that $\overline{G}^{M,*}(q_h) > \overline{G}^{M,*}(q_l)$ and $\alpha^{M,*} > 0$. This, in turn implies a profitable deviation to $\beta^S(q_l) = 1$. It follows that $\beta^{S,*}(q_j) = 0$ for some q_j cannot be supported as a *SPI* equilibrium and thus in any *SPI* equilibrium $\beta^{S,*}(q_j) > 0$ for all j.

To show that $\alpha^{M,*} < 1$, note that by eq. (12) $\overline{G}_E^{M,*}(q_h, q_U^{M,*}, 1) = \overline{G}^{M,*}(q_h) > \overline{G}^{S,*}(q_h) \geq \overline{G}_E^{S,*}(q_h, q_U^{S,*}, \alpha^{S,*})$. Therefore, $\alpha^{M,*} = 1$ results in $\beta^{S,*}(q_h) = 0$, contradicting property 1). Analogously, $\alpha^{S,*} = 1$ implies that $\overline{G}_E^{S,*}(q_l, q_U^{S,*}, 1) = \overline{G}^{S,*}(q_l) < \overline{G}^{M,*}(q_l) \leq \overline{G}_E^{M,*}(q_l, q_U^{M,*}, \alpha^{M,*})$, which in turn implies that $\beta^{S,*}(q_l) = 0$, contradicting property 1).

Finally, to establish that $\alpha^{S,*} > 0$, note that by Definition 1, $\alpha^{S,*} = 0$ implies that $\alpha^{M,*} > 0$ and $\beta^{M,*}(q_j) > 0$ for all j by eq. (9) and Lemma 3. Then, by Lemma 2 and eq. (12), $\overline{G}_E^{M,*}(q_h, q_U^{M,*}, \alpha^{M,*}) > \overline{G}_E^{M,*}(q_l, q_U^{M,*}, \alpha^{M,*})$ and $\overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*}) = \overline{G}^{S,*}(q_U^{S,*})$ for all j. Since by property 1) $\beta^{S,*}(q_j) > 0$ for all j, it must be true that $\overline{G}^{S,*}(q_U^{S,*}) \geq \overline{G}_E^{M,*}(q_h, q_U^{M,*}, \alpha^{M,*}) > \overline{G}_E^{M,*}(q_l, q_U^{M,*}, \alpha^{M,*})$, which implies that $\beta^{M,*}(q_l) = 0$, contradicting $\beta^{M,*}(q_j) > 0$ for all j. This completes the proof. ■

Proof of Proposition 5

To establish that $q_U^{S,*} > q_U^{M,*}$, by Lemma 2, it suffices to show that $\overline{G}^{S,*}(q_U^{S,*}) > \overline{G}^{M,*}(q_U^{M,*})$. By means of a contradiction, suppose that $\overline{G}^{S,*}(q_U^{S,*}) \leq \overline{G}^{M,*}(q_U^{M,*})$. By Lemma 4, $\beta^{S,*}(q_j) > 0$ for all j, which requires that $\overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*}) \geq \overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*})$. Let $\alpha^{M,*} = \alpha^{S,*} + \epsilon$. Then, $\overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*})$ can be re-written as

$$\overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*}) = \overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{S,*}) + \epsilon \left[\overline{G}^{M,*}(q_j) - \overline{G}^{M,*}(q_U^{M,*})\right]$$

Note that $\overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{S,*}) > \overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*})$ since $\overline{G}^{M,*}(q_U^{M,*}) \geq \overline{G}^{S,*}(q_U^{S,*})$ (by assumption), $\overline{G}^{M,*}(q_j) > \overline{G}^{S,*}(q_j)$ (by Lemma 2), and $\alpha^{S,*} > 0$ (by Lemma 4). Thus, $\overline{G}_E^{S,*}(q_h, q_U^{S,*}, \alpha^{S,*}) \geq \overline{G}_E^{M,*}(q_h, q_U^{M,*}, \alpha^{M,*})$ requires $\epsilon < 0$ since $\overline{G}^{M,*}(q_h) > \overline{G}^{M,*}(q_U^{M,*})$. This, however, results in $\overline{G}_E^{S,*}(q_l, q_U^{S,*}, \alpha^{S,*}) < \overline{G}_E^{M,*}(q_l, q_U^{M,*}, \alpha^{M,*})$ since $\overline{G}^{M,*}(q_l) < \overline{G}^{M,*}(q_U^{M,*})$, contradicting $\beta^{S,*}(q_l) > 0$. Therefore, $\overline{G}^{S,*}(q_U^{S,*}) \leq \overline{G}^{M,*}(q_U^{M,*})$ must hold in any *SPI* equilibrium. By Lemma 2, this implies that $q_U^{S,*} > q_U^{M,*}$. This, in turn, requires that $\pi_h^{S,*} > \pi_h^{M,*}$, which coupled with $\overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*}) \geq \overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*})$ for all j and $\overline{G}_E^{Z,*}(q_h, q_U^{Z,*}, \alpha^{Z,*}) > \overline{G}_E^{Z,*}(q_l, q_U^{Z,*}, \alpha^{Z,*})$ for all Z results in $\sum_j \pi_j^{S,*} \overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*}) > \sum_j \pi_j^{M,*} \overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*})$ for $j \in \{l, h\}$. ■

Proof of Proposition 6

1) is established in the proof of Lemma 4. To establish 2), note that by Lemma 3, $V_I^Z(\pi^Z)$ is continuous in π_h^Z and satisfies $V_I^Z(1,0) = V_I^Z(0,1) = 0$. Therefore, since $V_I^Z(\pi) > k$, there exist $\underline{\pi}_h^Z$ and $\overline{\pi}_h^Z$ satisfying $0 < \underline{\pi}_h^Z < \pi_h < \overline{\pi}_h^Z < 1$ such that $V_I^Z(\underline{\pi}_h^Z, 1 - \underline{\pi}_h^Z) = V_I^Z(\overline{\pi}_h^Z, 1 - \overline{\pi}_h^Z) = k$ for all Z. Let $\hat{\pi}_h^S = \overline{\pi}_h^S$ and $\hat{\pi}_h^M = \underline{\pi}_h^M$. Then, substituting for $\hat{\pi}_h^S$ and $\hat{\pi}_h^M$ in eq. (9) and taking into account that $\beta^M(q_j) = 1 - \beta^S(q_j)$ yields $\beta^{S,*}(q_j)$ for $j \in \{l, h\}$ given by eq. (16). Moreover, $0 < \beta^{S,*}(q_l) < \beta^{S,*}(q_h) < 1$ follows immediately from $\hat{\pi}_h^M < \pi_h < \hat{\pi}_h^S$. Lastly, we need to ensure that there are no deviation incentives from $\beta^{Z,*}(q_j)$, which requires $\overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*}) = \overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*})$ for all j. Solving for $\alpha^{M,*}$ and $\alpha^{S,*}$ yields:

$$\alpha^{M,*} = \frac{\overline{G}^{S,*}(q_U^{S,*}) - \overline{G}^{M,*}(q_U^{M,*})}{\overline{G}^{M,*}(q_h)\frac{\overline{G}^{S,*}(q_U^{S,*}) - \overline{G}^{S,*}(q_l)}{\overline{G}^{S,*}(q_h) - \overline{G}^{S,*}(q_l)} + \overline{G}^{M,*}(q_l)\frac{\overline{G}^{S,*}(q_h) - \overline{G}^{S,*}(q_U^{S,*})}{\overline{G}^{S,*}(q_h) - \overline{G}^{S,*}(q_l)} - \overline{G}^{M,*}(q_U^{S,*})} \in (0,1)$$

$$\alpha^{S,*} = \frac{\overline{G}^{S,*}(q_U^{S,*}) - \overline{G}^{M,*}(q_U^{M,*})}{\overline{G}^{S,*}(q_U^{S,*}) - \overline{G}^{S,*}(q_h)\frac{\overline{G}^{M,*}(q_U^{M,*}) - \overline{G}^{M,*}(q_l)}{\overline{G}^{M,*}(q_h) - \overline{G}^{M,*}(q_l)} - \overline{G}^{S,*}(q_l)\frac{\overline{G}^{M,*}(q_h) - \overline{G}^{M}(q_U^{M,*})}{\overline{G}^{M,*}(q_h) - \overline{G}^{M,*}(q_l)}} \in (0,1)$$

where $\alpha^{Z,*} \in (0,1)$ follows immediately from the following equilibrium property established by Lemma 2 and Proposition 5:

$$\overline{G}^{S,*}(q_l) < \overline{G}^{M,*}(q_l) < \overline{G}^{M}(\hat{q}_U^{M}) < \overline{G}^{S,*}(\hat{q}_U^{S}) < \overline{G}^{S,*}(q_h) < \overline{G}^{M,*}(q_h).$$

This completes the proof. ■

Proof of Lemma 5

First, we establish property 2). Note that if $\pi_j^{S,*} = 1$ for some j, then $\pi_{-j}^{S,*} = 0$ where $-j \in \{1, 2..., t\} \setminus \{j\}$ and $q_U^{S,*} = q_j$. Consequently, from eq. (17), $V_I^S(\pi^{S,*}) = 0$ with $\alpha^{S,*} = 0$ and $\alpha^{M,*} > 0$ (by Definition 1). Then, by eq. (12), $\overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*}) = \overline{G}^{S,*}(q_j)$ for all j. If $j = 1$, then there is a strict deviation incentive to $\beta^M(q_1) = 1$ since $\overline{G}^{S,*}(q_1) < \overline{G}^{M,*}(q_1) \leq \overline{G}_E^{M,*}(q_1, q_U^{M,*}, \alpha^{M,*})$. If $j > 1$, then $\beta^S(q_j) > 0$ implies that $\overline{G}^{S,*}(q_j) \geq \overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*}) > \overline{G}_E^{M,*}(q_{j-1}, q_U^{M,*}, \alpha^{M,*})$ due to the fact that $\overline{G}^{M,*}(q_j) > \overline{G}^{M,*}(q_{j-1})$ and $\alpha^{M,*} > 0$. This, in turn, implies a profitable deviation to $\beta^S(q_{j-1}) = 1$, contradicting $\pi_{j-1}^{S,*} = 0$ (by eq. 9). Thus, it follows that in any *SPI* equilibrium, $\pi_j^{S,*} < 1$ for all j.

To show that $\alpha^{M,*} < 1$, consider the contrary- $\alpha^{M,*} = 1$. Then, for any type q_j satisfying $\beta^S(q_j) > 1$ and $q_j \geq q_U^{S,*}$, eq. (12) and Lemma 2 imply that $\overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*}) = \overline{G}^{M,*}(q_j) > \overline{G}^{S,*}(q_j) \geq \overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*})$. This results in strict deviation incentives to $\beta^M(q_j) = 1$. Thus, in every *SPI* equilibrium, $\alpha^{M,*} < 1$.

Finally, to establish that $\alpha^{S,*} > 0$, by means of a contradiction suppose that $\alpha^{S,*} = 0$, which by Definition 1 implies that $\alpha^{M,*} > 0$. Consider type q_j such that $\beta^{S,*}(q_j) > 0$ and $q_j \geq q_U^{S,*}$. By Definition 1, the existence of such type is guaranteed. Note that $\alpha^{S,*} = 0$ and $\beta^{S,*}(q_j) > 0$ imply that $\overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*}) = \overline{G}^{S,*}(q_U^{S,*}) \geq \overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*})$. Moreover, since $\overline{G}^{Z,*}(q_j)$ is strictly increasing in q_j, $q_j \geq q_U^{S,*}$ implies that $\overline{G}^{S,*}(q_j) \geq \overline{G}^{S,*}(q_U^{S,*}) \geq \overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*})$. The last inequality requires $q_j > q_U^{M,*}$. To see this, note that by Lemma 2, $\overline{G}^{M,*}(q_j) > \overline{G}^{S,*}(q_j)$ and $q_j \leq q_U^{M,*}$ implies that $\overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*}) \geq \overline{G}^{M,*}(q_j) > \overline{G}^{S,*}(q_j)$, which contradicts the earlier inequality. Thus, $q_j > q_U^{M,*} = \sum_j \pi_j^{M,*} q_j$. This, in turn, implies the existence of a quality type $q_i < q_U^{M,*}$ with $\pi_i^{M,*} > 0$, which by eq. (9) requires $\beta^{M,*}(q_i) > 0$. However, $\alpha^{M,*} > 0$ implies that $\overline{G}_E^{M,*}(q_i, q_U^{M,*}, \alpha^{M,*}) < \overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*}) \leq \overline{G}^{S,*}(q_U^{S,*}) = \overline{G}_E^{S,*}(q_i, q_U^{S,*}, \alpha^{S,*})$, contradicting the optimality of $\beta^{M,*}(q_i) > 0$. Thus, $\alpha^{S,*} = 0$ and $\alpha^{M,*} > 0$ leads to a profitable deviation by some $q_i < q_U^{M,*}$ and thus cannot be supported in a *SPI* equilibrium. This proves that $\alpha^{S,*} > 0$ in every *SPI* equilibrium. ■

Proof of Proposition 7

Analogous to Proposition 5, in order to establish that $q_U^{S,*} > q_U^{M,*}$, by Lemma 2, it suffices to show that $\overline{G}^{S,*}(q_U^{S,*}) > \overline{G}^{M,*}(q_U^{M,*})$.[30] By a contradiction argument, suppose that $\overline{G}^{S,*}(q_U^{S,*}) \leq \overline{G}^{M,*}(q_U^{M,*})$. Since $\pi_j^{S,*} < 1$ for all j (Lemma 5), by eq. (9) there exist at least two quality types, denoted by $q_{\hat{l}}$ and $q_{\hat{h}}$, such that $\beta^{S,*}(q_j) > 0$ for all $j \in \{\hat{l}, \hat{h}\}$ and $q_{\hat{l}} < q_U^{S,*} < q_{\hat{h}}$. Moreover, for all $j \in \{\hat{l}, \hat{h}\}$, $\beta^{S,*}(q_j) > 0$ implies $\overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*}) \geq \overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*})$. If $q_U^{M,*} \geq q_{\hat{h}}$ then by Lemma 2, $\overline{G}^{M,*}(q_U^{M,*}) \geq \overline{G}^{M,*}(q_{\hat{h}}) > \overline{G}^{S,*}(q_{\hat{h}}) > \overline{G}^{S,*}(q_U^{S,*})$, which implies $\overline{G}_E^{M,*}(q_{\hat{h}}, q_U^{M,*}, \alpha^{M,*}) > \overline{G}_E^{S,*}(q_{\hat{h}}, q_U^{S,*}, \alpha^{S,*})$, a contradiction. Thus $q_{\hat{h}} > q_U^{M,*}$. If $q_U^{M,*} \leq q_{\hat{l}}$ then $\overline{G}^{M,*}(q_{\hat{l}}) \geq \overline{G}^{M,*}(q_U^{M,*}) \geq \overline{G}^{S,*}(q_U^{S,*}) > \overline{G}^{S,*}(q_{\hat{l}})$. Since by Lemma 5 $\alpha^{S,*} > 0$, this implies $\overline{G}_E^{M,*}(q_{\hat{l}}, q_U^{M,*}, \alpha^{M,*}) > \overline{G}_E^{S,*}(q_{\hat{l}}, q_U^{S,*}, \alpha^{S,*})$, a contradiction. Thus, $q_{\hat{l}} < q_U^{M,*} < q_{\hat{h}}$. Let $\alpha^{M,*} = \alpha^{S,*} + \epsilon$. Then, $\overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*})$ can be rewritten as

$$\overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{M,*}) = \overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{S,*}) + \epsilon \left[\overline{G}^{M,*}(q_j) - \overline{G}^{M,*}(q_U^{M,*}) \right]$$

From here, the proof is analogs to that of Proposition 5. The assumption, $\overline{G}^{M,*}(q_U^{M,*}) \geq \overline{G}^{S,*}(q_U^{S,*})$ and $\alpha^{S,*} > 0$ (by Lemma 5) imply $\overline{G}_E^{M,*}(q_j, q_U^{M,*}, \alpha^{S,*}) > \overline{G}_E^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*})$. Therefore, $\overline{G}_E^{S,*}(q_{\hat{h}}, q_U^{S,*}, \alpha^{S,*}) \geq \overline{G}_E^{M,*}(q_{\hat{h}}, q_U^{M,*}, \alpha^{M,*})$ requires $\epsilon < 0$. However, this results in $\overline{G}_E^{S,*}(q_{\hat{l}}, q_U^{S,*}, \alpha^{S,*}) < \overline{G}_E^{M,*}(q_{\hat{l}}, q_U^{M,*}, \alpha^{M,*})$, a contradiction. Thus, in any *SPI* equilibrium, $\overline{G}^{S,*}(q_U^{S,*}) > \overline{G}^{M,*}(q_U^{M,*})$. ■

Lemma A-3 *For $Z = N$, let $q_{\mathcal{L}}^N \in \{q_l, q_U^N, q_h\}$ denote the lead donor's quality type, $\overline{G}_\infty^{N,*}(q_{\mathcal{L}}^N, q_U^N, \alpha^N) = \lim_{n\to\infty} \overline{G}^{N,*}(q_{\mathcal{L}}^N, q_U^N, \alpha^N)$ denote the total contributions in the limit economy given $q_{\mathcal{L}}^N$ and the posterior expected quality q_U^N, and $C_\infty^N = \lim_{n\to\infty} C^N$ denote the equilibrium contributor's set in the limit economy. Then, in the limit economy ($n \to \infty$):*

a) any donor with $w_i < w_1$ is a non-contributor, i.e. $i \notin C_\infty^N$ for any $i > 1$;

b) the total equilibrium donations satisfy $\overline{G}_\infty^{S,*}(q_l) \leq \overline{G}_\infty^{N,*}(q_l, q_U^N, \alpha^N) \leq \overline{G}_\infty^{N,*}(q_h, q_U^N, \alpha^N) \leq \overline{G}_\infty^{S,*}(q_h)$, with strict inequalities for $q_U^N \in (q_l, q_h)$. Moreover, $\overline{G}_\infty^{S,*}(q_l) = \overline{G}_\infty^{N,*}(q_l, q_l, \alpha^N)$ and $\overline{G}_\infty^{S,*}(q_h) = \overline{G}_\infty^{N,*}(q_h, q_h, \alpha^N)$.

Proof of Lemma A-3

Analogous to $Z = \{S, M\}$, let $\eta_{\mathcal{L}}^N$ denote the posterior belief by the follower donors of type $q_{\mathcal{L}}^N$ upon observing $Z = N$. Then, given a conjecture about others' contributions $\overline{G}_{-i}^N$, a contributing donor i maximizes

$$E_{q_{\mathcal{L}}^N}[u_i(g_i, \overline{G}_{-i}^N, q_{\mathcal{L}}^N)] = h(w_i - g_i) + \sum_{\mathcal{L}} \eta_{\mathcal{L}}^N q_{\mathcal{L}}^N v\left(\overline{G}_{-i}^N + g_i\right) \quad \text{(A-14)}$$

[30] It is straightforward to establish that Lemma 2 extends to the case of multiple quality types.

Therefore, i's giving for $Z = N$ satisfies[31]

$$h'(w_i - \overline{g}_i^{N,*}) = \sum_{\mathcal{L}} \eta_{\mathcal{L}}^N q_{\mathcal{L}}^N v' \left(\overline{G}^{N,*}(q_{\mathcal{L}}^N, q_U^N, \alpha^N) \right) \tag{A-15}$$

Analogous to $Z = \{S, M\}$, $\overline{g}_i^{N,*}(q_U^N, \alpha^N)$ is (weakly) increasing in w_i. Therefore, $i \in C^N$, implies that $i-1 \in C^N$. To establish a), note that $i \in C_\infty^N$ for $i > 1$, implies that $\overline{g}_{1,\infty}^{N,*}(q_U^N, \alpha^N) > 0$ and $\overline{G}_\infty^{N,*}(q_{\mathcal{L}}, q_U^N, \alpha^N) \geq \lim_{n\to\infty} nt_1 \overline{g}_{1,\infty}^{N,*}(q_U^N, \alpha^N) = \infty$. This, in turn, leads to a contradiction since $\lim_{n\to\infty} qv'(G) = 0$ and by eq. (A-15) this implies that $\overline{g}_{i,\infty}^{N,*}(q_U^N, \alpha^N) = 0$ for all i and thus $C_\infty^N = \emptyset$. Therefore, $i \notin C_\infty^N$ for any $i > 1$.

To establish part b), note that $\overline{G}_\infty^{N,*}(q_{\mathcal{L}}, q_U^N, \alpha^N) = \overline{G}_{F,\infty}^{N,*}(q_U^N, \alpha^N) + \overline{g}_{L,\infty}^{N,*}(q_{\mathcal{L}}, q_U^N, \alpha^N)$, where the first term denotes the follower donors' total contributions, which are independent of the unobservable type $q_{\mathcal{L}}$, and the second term denotes the lead donor's contribution. If $1 \in C_\infty^N$, then analogous to the argument above, $\lim_{n\to\infty} \overline{g}_{1,\infty}^{N,*}(q_U^N, \alpha^N) = 0$, which by eq. (A-15) implies the following equilibrium condition:

$$\begin{aligned} h'(w_1) &= \alpha^N \left(\pi_l^N q_l v'(\overline{G}_\infty^{N,*}(q_l, q_U^N, \alpha^N)) + \pi_h^N q_h v'(\overline{G}_\infty^{N,*}(q_h, q_U^N, \alpha^N)) \right) + \\ &+ (1-\alpha^N) q_U^N v'(\overline{G}_\infty^{N,*}(q_U^N, q_U^N, \alpha^N)), \end{aligned} \tag{A-16}$$

where we have used eq. (10) to substitute for $\eta_{\mathcal{L}}^N$. Since the lead donor's contribution is unobservable for $Z = N$, $\overline{g}_{L,\infty}^{N,*}(q_{\mathcal{L}}^N, q_U^N, \alpha^N)$ maximizes eq. (1) resulting in

$$h'(w_1 - \overline{g}_{L,\infty}^{N,*}(q_{\mathcal{L}}^N, q_U^N, \alpha^N)) \geq q_{\mathcal{L}}^N v'(\overline{G}_\infty^{N,*}(q_{\mathcal{L}}^N, q_U^N, \alpha^N)), \tag{A-17}$$

where the inequality in eq. (A-17) is strict if and only if $h'(w_1) > q_{\mathcal{L}}^N v'(\overline{G}_\infty^{N,*}(q_{\mathcal{L}}^N, q_U^N, \alpha^N))$. Moreover, by eq. (A-17), $q_{\mathcal{L}}^N v'(\overline{G}_{F,\infty}^N(q_U^N, \alpha^N) + \overline{g}_L^N) > \hat{q}_{\mathcal{L}}^N v'(\overline{G}_{F,\infty}^N(q_U^N, \alpha^N) + \overline{g}_L^N)$ for any $q_{\mathcal{L}}^N > \hat{q}_{\mathcal{L}}^N$ and thus $h''(\cdot) < 0$ implies that $\overline{g}_{L,\infty}^{N,*}(q_{\mathcal{L}}^N, q_U^N, \alpha^N) \geq \overline{g}_{L,\infty}^{N,*}(\hat{q}_{\mathcal{L}}^N, q_U^N, \alpha^N)$ and $\overline{G}_\infty^{N,*}(q_{\mathcal{L}}^N, q_U^N, \alpha^N) \geq \overline{G}_\infty^{N,*}(\hat{q}_{\mathcal{L}}^N, q_U^N, \alpha^N)$, with strict inequality if $\overline{g}_{L,\infty}^{N,*}(q_{\mathcal{L}}^N, q_U^N, \alpha^N) > 0$.

To compare $\overline{G}_\infty^{N,*}(q_j, q_U^N, \alpha^N)$ and $\overline{G}_\infty^{S,*}(q_j)$ for $j = \{l, h\}$, recall from eq. (A-7) that $\overline{G}_\infty^{S,*}(q_j) = G_1^{S,0}(q_j)$, where $G_1^{S,0}(q_j)$ solves eq. (3). Since $h''(\cdot) < 0$, by eq. (3) and eq. (A-17), it follows that $\overline{g}_{L,\infty}^{N,*}(q_j, q_U^N, \alpha^N) > 0$ implies

$$q_j v'(\overline{G}_\infty^{N,*}(q_j, q_U^N, \alpha^N)) = h'(w_1 - \overline{g}_{L,\infty}^N(q_j, q_U^N, \alpha^N)) > h'(w_1) = q_j v'(\overline{G}_\infty^{S,*}(q_j)). \tag{A-18}$$

Since $v''(\cdot) < 0$, the above inequality results in $\overline{G}_\infty^{N,*}(q_j, q_U^N, \alpha^N) < \overline{G}_\infty^{S,*}(q_j)$.

To establish that $\overline{G}_\infty^{N,*}(q_h, q_U^N, \alpha^N) < \overline{G}_\infty^{S,*}(q_h)$ for $q_U^N < q_h$, by (A-18) it suffices to show that $\overline{g}_{L,\infty}^{N,*}(q_h, q_U^N, \alpha^N) > 0$. Since $\overline{g}_{L,\infty}^{N,*}(q_h, q_U^N, \alpha^N) \geq \overline{g}_{L,\infty}^{N,*}(q_e, q_U^N, \alpha^N)$ for $e \in \{l, U\}$, $\overline{g}_{L,\infty}^{N,*}(q_h, q_U^N, \alpha^N) =$

[31] Note that there is a one-to-one relationship between π_h^N and q_U^N, allowing us to express the optimal equilibrium contributions $\overline{G}_\infty^{N,*}(q_j, q_U^N, \alpha^N)$ as a function of q_U^N.

0 implies by eq. (A-16) that $\overline{G}_\infty^{N,*}(q_\mathcal{L}, q_U^N, \alpha^N)$ is independent of $q_\mathcal{L}$ and α^N and solves $h'(w_1) = q_U^N v'(\overline{G}_\infty^{N,*}(q_U^N))$ for all $q_\mathcal{L}$. However, by eq. (A-17), this results in a contradiction since $h'(w_1) < q_h v'(\overline{G}_\infty^{N,*}(q_U^N))$ for $q_U^N < q_h$ implies a profitable deviation to $\overline{g}_{L,\infty}^N(q_h, q_U^N, \alpha^N) > 0$. Therefore, for $q_U^N < q_h$, $\overline{g}_{L,\infty}^N(q_h, q_U^N, \alpha^N) > 0$ and by (A-18), $\overline{G}_\infty^{N,*}(q_h, q_U^N, \alpha^N) < \overline{G}_\infty^{S,*}(q_h)$. For $q_U^N = q_h$, $\overline{g}_{L,\infty}^{N,*}(q_h, q_h, \alpha^N) = 0$ and $h'(w_1) = q_h v'(\overline{G}_\infty^{N,*}(q_h, q_h, \alpha^{N,*}))$ satisfies eqs. (A-16) and (A-17), which by (A-18) results in $\overline{G}_\infty^{N,*}(q_h, q_U^N, \alpha^N) = \overline{G}_\infty^{S,*}(q_h)$.

To establish that $\overline{G}_\infty^{N,*}(q_l, q_U^N, \alpha^N) > \overline{G}_\infty^{S,*}(q_l)$ for $q_U^N > q_l$, note that by eq. (3) and (A-18), $\overline{G}_\infty^{N,*}(q_l, q_U^N, \alpha^N) \leq \overline{G}_\infty^{S,*}(q_l)$ requires that $q_l v'(\overline{G}_\infty^{N,*}(q_l, q_U^N, \alpha^N)) = h'(w_1 - \overline{g}_{L,\infty}^N(q_l, q_U^N, \alpha^N))$. Since $\overline{g}_{L,\infty}^{N,*}(q_\mathcal{L}, q_U^N, \alpha^N)$ is increasing in $q_\mathcal{L}$, this implies that eq. (A-17) holds with equality for all $q_\mathcal{L}$. Substituting for eq. (A-17) into eq. (A-16) results in

$$\begin{aligned} h'(w_1) &= \alpha^N \left(\pi_l^N q_l h'(w_1 - \overline{g}_{L,\infty}^{N,*}(q_l, q_U^N, \alpha^N)) + \pi_h^N h'(w_1 - \overline{g}_{L,\infty}^{N,*}(q_h, q_U^N, \alpha^N)) \right) + \\ &+ (1 - \alpha^N) h'(w_1 - \overline{g}_{L,\infty}^{N,*}(q_U^N, q_U^N, \alpha^N)). \end{aligned}$$

For $q_U^N > q_l$, requiring $\pi_h^N > 0$, the above equation is clearly violated since $\overline{g}_{L,\infty}^{N,*}(q_h, q_U^N, \alpha^N) > \overline{g}_{L,\infty}^{N,*}(q_U^N, q_U^N, \alpha^N) > 0$. Therefore, for $q_U^N > q_l$, $\overline{G}_\infty^{N,*}(q_l, q_U^N, \alpha^N) \leq \overline{G}_\infty^{S,*}(q_l)$ cannot be supported in equilibrium, implying that $\overline{G}_\infty^{N,*}(q_l, q_U^N, \alpha^N) > \overline{G}_\infty^{S,*}(q_l)$. For $q_U^N = q_l$, $\pi_h^N = 0$ and $\overline{g}_{L,\infty}^{N,*}(q_U^N, q_U^N, \alpha^N) = \overline{g}_{L,\infty}^{N,*}(q_l, q_U^N, \alpha^N) = 0$ solve eq. (A-16) and eq. (A-17) with equalities, resulting in $\overline{G}_\infty^{N,*}(q_l, q_l, \alpha^N) = \overline{G}_\infty^{S,*}(q_l)$.

Finally, to complete the proof, we establish that $1 \notin C_\infty^N$ cannot be sustained in equilibrium. In that case, by part a), $C_\infty^N = \emptyset$, which in turn implies $\overline{G}_{F,\infty}^{N,*}(q_U^N, \alpha^N) = 0$. Then, eq. (A-17) holds with equality for all $q_\mathcal{L}$ and thus $\overline{g}_{L,\infty}^{N,*}(q_\mathcal{L}) > 0$ for all $q_\mathcal{L}$ since $h'(w_1) > q_\mathcal{L} v'(0)$. It follows that $h'(w_1) < q_\mathcal{L} v'(\overline{G}_\infty^{N,*}(q_\mathcal{L}))$ for all $q_\mathcal{L}$, which by eq. (A-15) implies that $\overline{g}_{1,\infty}^N(q_U^N, \alpha^N) > 0$, contradicting $C_\infty^N = \emptyset$. Therefore, $1 \in C_\infty^N$. ■

Proof of Lemma 6

To prove that $\beta^{S,*}(q_j) = 0$ for all j, analogous to the proof of Proposition 4, suppose by means of a contradiction that $\beta^{S,*}(q_j) > 0$ for some j. This, in turn, implies that $\beta^{S,*}(q_j) > 0$ for all j since $\alpha^{S,*} = 1$ requires $V_I^S(\pi^{S,*}) \geq k > 0$, which by Lemma 3 necessitates $\pi_h^{S,*} \in (0,1)$. Moreover, by eq. (12), $\overline{G}_{E,\infty}^{S,*}(q_l, q_U^{S,*}, 1) = \overline{G}_\infty^{S,*}(q_l)$. However, this results in a profitable deviation to $\beta^S(q_l) = 0$ since $\overline{G}_\infty^{S,*}(q_l) < \overline{G}_\infty^{M,*}(q_l) \leq \overline{G}_{E,\infty}^{M,*}(q_l, q_U^{M,*}, \alpha^{M,*})$. Therefore, $\beta^{S,*}(q_j) > 0$ for some j cannot be supported in equilibrium, implying that $\beta^{S,*}(q_j) = 0$ for all j.

To complete the proof, note that $\beta^{M,*}(q_j) > 0$ for some q_j, requires $\alpha^{M,*} = 1$ in a fully informed equilibrium. Then, by eq. (12) and Lemmas 2 and A-3, $\overline{G}_{E,\infty}^{M,*}(q_h, q_U^{M,*}, 1) = \overline{G}_\infty^{M,*}(q_h) > \overline{G}_\infty^{S,*}(q_h) \geq \overline{G}_\infty^{N,*}(q_h, q_U^{N,*}, 1)$, implying that $\beta^{N,*}(q_h) = 0$. Then, if $\beta^{N,*}(q_l) > 0$, it follows by eq. (9) that $\pi_l^{N,*} = 1$. By Lemmas 2 and A-3 this implies that $\overline{G}_\infty^{N,*}(q_l, q_l, 1) = \overline{G}_\infty^{S,*}(q_l) < \overline{G}_\infty^{M,*}(q_l)$, contradicting $\beta^{N,*}(q_l) > 0$. Therefore, $\beta^{M,*}(q_j) > 0$ for some q_j implies $\beta^{N,*}(q_j) = 0$ for all q_j. Conversely, $\beta^{N,*}(q_j) > 0$ for some q_j requires $\beta^{M,*}(q_j) = 0$. Since $\beta^{S,*}(q_j) = 0$ for all j, this implies that $\beta^{N,*}(q_j) = 1$ for all j. Such equilibrium is possible as long as $\overline{G}_{\infty}^{N,*}(q_l, q_U, 1) > \overline{G}^{M,*}(q_l)$ and is supported with an off-equilibrium belief of $\pi_h^{M,*} = 0$. ■

Proof of Proposition 8

Proving that $q_U^{S,*} > q_U^{M,*}$ is analogous to the proof of Proposition 5. Similar to Lemma 4, we first establish that $\beta^{S,*}(q_j) > 0$ for all q_j and $\alpha^{S,*} > 0$. In particular, $\beta^{S,*}(q_j) > 0$ follows from the fact that $\beta^{S,*}(q_j) = 0$ for some j implies that $V_I^S(\pi^{S,*}) = 0$, resulting in $\alpha^{S,*} = 0$. This, in turn, implies $q_U^{S,*} = q_{-j}$ where $q_{-j} = \{l, h\} \setminus \{j\}$ and by eq. (12), $\overline{G}_{E,\infty}^{S,*}(q_{-j}, q_{-j}, 0) = \overline{G}_{\infty}^{S,*}(q_{-j})$. If $-j = l$, then there is a profitable deviation to $\beta^{S,*}(q_l) = 0$ since $\overline{G}_{\infty}^{S,*}(q_l) < \overline{G}_{\infty}^{M,*}(q_l) \leq \overline{G}_{E,\infty}^{M,*}(q_l, q_U^{M,*}, \alpha^{M,*})$. If $-j = h$, then $\beta^{S,*}(q_h) > 0$ implies that $\overline{G}_{\infty}^{S,*}(q_h) \geq \overline{G}_{E,\infty}^{\tilde{Z},*}(q_h, q_U^{\tilde{Z},*}, \alpha^{\tilde{Z},*}) > \overline{G}_{E,\infty}^{\tilde{Z},*}(q_l, q_U^{\tilde{Z},*}, \alpha^{\tilde{Z},*})$ for $\tilde{Z} = \{M, N\}$ since $\overline{G}_{\infty}^{\tilde{Z},*}(q_h) > \overline{G}_{\infty}^{\tilde{Z},*}(q_l)$ for all $\tilde{Z}$. This, in turn implies a profitable deviation to $\beta^S(q_l) = 1$. Consequently, in any *SPI* equilibrium $\beta^{S,*}(q_j) > 0$ for all $j = \{l, h\}$.

To establish $\alpha^{S,*} > 0$, note that $\alpha^{S,*} = 0$ implies by eq. (12) that $\overline{G}_{E,\infty}^{S,*}(q_j, q_U^{S,*}, \alpha^{S,*}) = \overline{G}_{\infty}^{S,*}(q_U^{S,*})$ for all $j = \{l, h\}$. Moreover by Definition 1, $\alpha^{\tilde{Z},*} > 0$ for some $\tilde{Z}$, implying that $\beta^{\tilde{Z},*}(q_j) > 0$ for all j. However, $\overline{G}^{S,*}(q_U^{S,*}) \geq \overline{G}_{E,\infty}^{\tilde{Z},*}(q_h, q_U^{\tilde{Z},*}, \alpha^{\tilde{Z},*}) > \overline{G}_{E,\infty}^{\tilde{Z},*}(q_l, q_U^{\tilde{Z},*}, \alpha^{\tilde{Z},*})$. The last strict inequality implies a profitable deviation to $\beta^S(q_l) = 1$, implying that $\alpha^{S,*} = 0$ cannot be supported in equilibrium. Consequently, in any *SPI* equilibrium $\alpha^{S,*} > 0$. Given $\alpha^{S,*} > 0$ and $\beta^{S,*}(q_j) > 0$ for all $j = \{l, h\}$, establishing that $q_U^{S,*} > q_U^{M,*}$ is identical to the proof of Proposition 5 and thus omitted here.

To establish the existence of a *SPI* equilibrium with $q_U^{S,*} > \max\{q_U^{N,*}, q_U^{M,*}\}$, consider the equilibrium constructed in the proof of Lemma 4 with $\beta^{N,*}(q_j) = 0$ for all j and an off-equilibrium belief of $\pi_h^{N,*} = 0$, implying $q_U^{N,*} = q_l$. Then, it is straightforward to verify that $V_I^N(0, 1) = 0$, resulting in $\alpha^{N,*} = 0$. Then, by Lemma A-3, $\overline{G}_{\infty}^{N,*}(q_l, q_l, 0) = \overline{G}_{\infty}^{S,*}(q_l) \leq \overline{G}_{E,\infty}^{Z,*}(q_j, q_U^{Z,*}, \alpha^{Z,*})$ for $Z = \{S, M\}$ implying no profitable deviation to N. ■

Proof of Proposition 9

Let $\tilde{G}_{\infty}^{Z,*}(q) = \lim_{n\to\infty} \tilde{G}^{Z,*}(q)$. To establish part a), we first show that $\lim\limits_{\tilde{G}\to\infty} qv_g(G, 0) < h'(w_1)$ is necessary and sufficient for the existence of $\tilde{G}_i^{Z,0}(q, m\mathbb{1}_M) < \infty$ for all i that solves eq. (20). Recall that $v_{GG}(\cdot) < 0$, and $v_{GG}(\cdot) + v_{gG}(\cdot) < 0$, resulting in $qv_{GG}(G, 0)\,(1 + m\mathbb{1}_M) + qv_{gG}(G, 0) < 0$. Then, since $h''(\cdot) < 0$, implicit differentiation of eq. (20) results in $\frac{\partial \tilde{G}_i^{Z,0}(q, m\mathbb{1}_M)}{\partial w_i} = \frac{h''(w_i)}{qv_{GG}(\tilde{G}_i^{Z,0}, 0)(1+m\mathbb{1}_M) + qv_{gG}(\tilde{G}_i^{Z,0}, 0)} > 0$. Thus, it suffices to establish that $\tilde{G}_1^{Z,0}(q, m\mathbb{1}_M) < \infty$. Note that $qv_G(0, 0)\,(1 + m\mathbb{1}_M) + qv_g(0, 0) - h'(w_1) > 0$ since $qv_G(0, 0) > h'(w_1)$. Then, $qv_{GG}(G, 0)\,(1 + m\mathbb{1}_M) + qv_{gG}(G, 0) < 0$ implies that there is at most one value $\tilde{G}_1^{Z,0}(q, m\mathbb{1}_M)$ that satisfies eq. (20). Moreover, $\tilde{G}_1^{Z,0}(q, m\mathbb{1}_M) < \infty$ exists if an only if the left-hand side of eq. (20) turns strictly negative for some value of G, i.e. $\lim_{G\to\infty} qv_G(G,0)\,(1+m\mathbb{1}_M) + qv_g(G,0) - h'(w_1) < 0$. Since by assumption, $\lim_{G\to\infty} qv_G(G,0) = 0$, this condition reduces to $\lim_{G\to\infty} qv_g(G,0) - h'(w_1) < 0$.

Given $\tilde{G}_i^{Z,0}(q, m\mathbb{1}_M) < \infty$ for all i, proving that $\tilde{G}_\infty^{S,*}(q) \leq \tilde{G}_\infty^{M,*}(q)$ is analogous to the proof of Proposition 2. First, by a symmetric argument to the proof of Lemma A-1, it is straightforward to establish that $\lim_{n\to\infty} \tilde{G}^{Z,L}(q, d_L^Z) \to \tilde{G}_1^{Z,0}(q, m\mathbb{1}_M)$. Moreover, note that by eq. (20), $\tilde{G}_i^{M,0}(q,0) = \tilde{G}_i^{S,0}(q)$ and implicit differentiation of the same equation results in

$$\frac{d\tilde{G}_i^{M,0}(q,m)}{dm} = \frac{\tilde{v}_G\left(\tilde{G}_i^{M,0}(q,m),0\right)}{-\left(\tilde{v}_{GG}\left(\tilde{G}_i^{M,0}(q,m),0\right)(1+m) + \tilde{v}_{gG}\left(\tilde{G}_i^{M,0}(q,m),0\right)\right)} > 0, \tag{A-19}$$

since $qv_{GG}(G,0)\,(1+m\mathbb{1}_M) + qv_{gG}(G,0) < 0$. Thus, $\tilde{G}_\infty^{S,*}(q) \leq \tilde{G}_\infty^{M,*}(q,m)$, with strict inequality whenever $m > 0$. To show that $\tilde{m}^*(q) > 0$, analogous to the proof of Proposition 2, it suffices to show that $\frac{d\tilde{u}_L(q,0)}{dm} > 0$. The objective function of the lead donor for $Z = M$ is given by

$$\tilde{u}_L(q,m) = h(w_1 - \tilde{G}^{Z,L}(q,m) + \tilde{G}_F^Z(q,m)) + q\tilde{v}\left(\tilde{G}^{Z,L}(q,m), \tilde{G}^{Z,L}(q,m) - \tilde{G}_F^Z(q,m)\right) \tag{A-20}$$

where $\tilde{G}_F^Z(q,m)$ denotes the follower donors' aggregate best response for a fixed m.[32] In order for the lead donor to choose $m > 0$, it must be true that $\lim_{n\to\infty} \frac{d\tilde{u}_L(q,0)}{dm} > 0$. Recall that for $m = 0$, $\tilde{G}_1^{M,0}(q,0) = \tilde{G}_F^M(q,0) = \tilde{G}_1^{S,0}(q)$. Thus, differentiating (A-20) w.r.t. m and evaluating it at $m = 0$ gives rise to

$$\begin{aligned}\lim_{n\to\infty} \frac{d\tilde{u}_L(q,0)}{dm} &= \left[-h'(w_1) + q\tilde{v}_G\left(\tilde{G}_1^{S,0}(q),0\right) + q\tilde{v}_g\left(\tilde{G}_1^{S,0}(q),0\right)\right]\frac{d\tilde{G}_1^{M,0}(q)}{dm} + \\ &+ \left[h'(w_1) + q\tilde{v}_g\left(\tilde{G}_1^{S,0}(q),0\right)\right]\lim_{n\to\infty}\frac{d\tilde{G}_F^M(q,0)}{dm}\end{aligned} \tag{A-21}$$

By eq. (20), the first term in the above equation equals 0. Moreover, since $h'(\cdot) > 0$ and $\tilde{v}_g(\cdot) > 0$, $\lim_{n\to\infty} \frac{d\tilde{u}_L(q,0)}{dm} > 0$ if and only if $\lim_{n\to\infty} \frac{d\tilde{G}_F^M(q,0)}{dm} > 0$. Since $\lim_{n\to\infty} \tilde{G}_F^M(q,m) = \frac{\tilde{G}_1^{M,0}(q,m)}{1+m}$, differentiating w.r.t. m gives $\lim_{n\to\infty} \frac{d\tilde{G}_F^M(q,m)}{dm} = \frac{1}{1+m}\frac{d\tilde{G}_1^{M,0}(q,m)}{dm} - \frac{\tilde{G}_1^{M,0}(q,m)}{(1+m)^2}$. Substituting for $\frac{d\tilde{G}_1^{M,0}(q,m)}{dm}$ from eq. (A-19) and evaluating at $m = 0$ obtains

$$\lim_{n\to\infty} \frac{d\tilde{G}_F^M(q,0)}{dm} = \tilde{G}_1^{S,0}(q)\left[\frac{\tilde{v}_G\left(\tilde{G}_1^{S,0}(q),0\right)}{-\left(\tilde{v}_{GG}\left(\tilde{G}_1^{S,0}(q),0\right) + \tilde{v}_{gG}\left(\tilde{G}_1^{S,0}(q),0\right)\right)\tilde{G}_1^{S,0}(q)} - 1\right] \tag{A-22}$$

[32] Similar to the base model, taking into account that $\tilde{G}_i^{Z,0}(q, m\mathbb{1}_M) < \infty$ for all i and that the individual best response for a contributing follower, $\tilde{g}_i^{Z,*}(q,G)$, uniquely solves eq. (19), we can employ the Andreoni-McGuire algorithm to uniquely pin down $\tilde{G}_F^Z(q, d_L^Z)$ (see Yildirim, 2014).

From the above equation, it is immediately evident that $\lim_{n\to\infty} \frac{d\tilde{u}_L(q,0)}{dm} > 0$ if and only if $\frac{-\left(\tilde{v}_{GG}(\tilde{G}_1^{S,0}(q),0)+\tilde{v}_{gG}(\tilde{G}_1^{S,0}(q),0)\right)\tilde{G}_1^{S,0}(q)}{\tilde{v}_G(\tilde{G}_1^{S,0}(q),0)} = \epsilon_{\tilde{v}}\left(\tilde{G}_1^{S,0}(q),0\right) < 1$, implying that $\tilde{m}^*(q) > 0$ and $\tilde{G}_\infty^{S,*}(q) < \tilde{G}_\infty^{M,*}(q)$.

To establish part b), note that from the proof of part a), $\lim\limits_{G\to\infty} qv_g(G,0) - h'(w_1) \geq 0$ implies that $\tilde{G}_1^{Z,0}(q, m\mathbb{1}_M) = \infty$. Since $\tilde{G}_i^{Z,0}(q,\mathbb{1}_M) \leq \tilde{G}_1^{Z,0}(q,\mathbb{1}_M)$ for all i, it must be the case that $1 \in C$. Otherwise, if $1 \notin C$, then $C = \emptyset$, implying that $\tilde{G}_\infty^{Z,*}(q) = g_L^{Z,*} \leq \infty$. This, in turn, implies that the individual contribution by the wealthiest follower type, $\tilde{g}_1^{Z,*}(q)$, must satisfy $\lim_{n\to\infty} \tilde{g}_1^{Z,*}(q) > 0$ since $\tilde{G}_1^{Z,0}(q,\mathbb{1}_M) > \tilde{G}_\infty^{Z,*}(q)$, contradicting $1 \notin C$. Given $1 \in C$, if $\tilde{G}_\infty^{Z,*}(q) < \infty$, then analogous to the above argument, $\lim_{n\to\infty} \tilde{g}_1^{Z,*}(q) > 0$, which in turn results in a contradiction since $\tilde{G}_\infty^{Z,*}(q) \geq \lim_{n\to\infty} nt_1\tilde{g}_1^{Z,*}(q) = \infty$. Therefore, this establishes that $\tilde{G}_\infty^{Z,*}(q) = \infty$ if $\lim\limits_{G\to\infty} qv_g(G,0) - h'(w_1) \geq 0$ and completes the proof. ■

Proof of Proposition 10

First, note that the lead donor's informed payoff is $\overline{u}_{L,\infty}^S(q_j,\gamma) = \overline{u}_L(q_j,q_j,\overline{g}_{L,\infty}^{S,*}(q_j))$, while the her uninformed payoff can be expressed as

$$\overline{u}_{L,\infty}^S(q_U^S,\gamma) = \overline{u}_L(q_U^S,q_U^S,\overline{g}_{L,\infty}^{S,*}(q_U^S)) + \gamma\left(\pi_h^S q_h v(G_1^{S,0}(q_h)) + \pi_l^S q_l v(G_1^{S,0}(q_l)) - q_U^S v(G_1^{S,0}(q_U^S))\right).$$

Therefore, taking into account that $q_U^S = \sum_j \pi_j^S q_j$ for $j \in \{l,h\}$, analogous to eq. (A-13), $V_{I,\infty}^S(\pi^S,\gamma)$ can be expressed as

$$V_{I,\infty}^S(\pi^S,\gamma) = \sum_{j=\{l,h\}} \pi_j^Z\left[\overline{u}_L(q_j,q_j,\overline{g}_{L,\infty}^{S,*}(q_j)) - \overline{u}_L(q_j,q_U^S,\overline{g}_{L,\infty}^{S,*}(q_U^S)) - \gamma q_j\left(v(G_1^{S,0}(q_j)) - v(G_1^{S,0}(q_U^S))\right)\right] \quad \text{(A-23)}$$

To account for the incentives constraints in eq. (11), recall from Proposition 1 that $\frac{dG_1^{S,0}(q)}{dg_L^S} = 0$ and thus if q is common knowledge, $g_{L,\infty}^S(q) = \lim_{n\to\infty} g_L^{S,*}(q) = 0$ for all q. Thus, the incentives constraints must be binding with $\overline{g}_{L,\infty}^S(q_\mathcal{L}) = \lim_{n\to\infty} \overline{g}_L^{S,*}(q_\mathcal{L}) > 0$ for $q_\mathcal{L} \in \{q_U^S, q_h\}$ where $q_U^S > q_l$.[33]. These binding constraints for the uninformed and the high type of lead donor are given by

$$\overline{u}_L(q_l,q_l,\overline{g}_{L,\infty}^{S,*}(q_l)) = \overline{u}_L(q_l,q_U^S,\overline{g}_{L,\infty}^{S,*}(q_U^S)) + \gamma q_l\left(v(G_1^{S,0}(q_l)) - v(G_1^{S,0}(q_U^S))\right) \quad \text{(A-24)}$$

$$\overline{u}_L(q_U^S,q_U^S,\overline{g}_{L,\infty}^{S,*}(q_U^S)) = \overline{u}_L(q_U^S,q_h,\overline{g}_{L,\infty}^{S,*}(q_h)) - \gamma q_U^S\left(v(G_1^{S,0}(q_h)) - v(G_1^{S,0}(q_U^S))\right) \quad \text{(A-25)}$$

Note that eq. (A-24) implies that the summation in eq. (A-23) for $j = l$ reduces to 0. To simplify the remaining expression, note that by definition,

$$\overline{u}_L(q_h,q_U^S,\overline{g}_{L,\infty}^{S,*}(q_U^S)) = \overline{u}_L(q_U^S,q_U^S,\overline{g}_{L,\infty}^{S,*}(q_U^S)) + (q_h - q_U^S)v(G_1^{S,0}(q_U^S)). \quad \text{(A-26)}$$

[33] Recall that $q_U^S = q_l$ implies that $(\pi_l^S,\pi_h^S) = (1,0)$ and thus $V_{I,\infty}^S((1,0),\gamma) = 0$.

Substituting for the incentive constraint given by eq. (A-25) in eq. (A-26) and taking into account that $\overline{u}_L(q_U^S, q_h, \overline{g}_{L,\infty}^{S,*}(q_h)) = \overline{u}_L(q_h, q_h, \overline{g}_{L,\infty}^{S,*}(q_h)) - (q_h - q_U^S)v(G_1^{S,0}(q_h))$, results in

$$\overline{u}_L(q_h, q_U^S, \overline{g}_{L,\infty}^{S,*}(q_U^S)) = \overline{u}_L(q_h, q_h, \overline{g}_{L,\infty}^{S,*}(q_h)) - \left(q_h - (1-\gamma)q_U^S\right)\left(v(G_1^{S,0}(q_h)) - v(G_1^{S,0}(q_U^S))\right). \quad \text{(A-27)}$$

Finally, substituting for $\overline{u}_L(q_h, q_U^S, \overline{g}_{L,\infty}^{S,*}(q_U^S))$ given by the above equation in eq. (A-23) and simplifying results in $V_{I,\infty}^S(\pi^S, \gamma) = (1-\gamma)(q_h - q_U^S)(v(G_1^{S,0}(q_h)) - v(G_1^{S,0}(q_U^S)))$, which completes the proof. ■